1. (11 pts) *As always, give answers in simplified exact form.*

(a) Consider the vector function $\mathbf{r}(t) = \langle 6 + t\,,\ 2\tan^{-1}(t)\,,\ 3t + e^{t^2}\,\rangle$.
Find the tangential component of acceleration at $t = 0$.

$$\vec{r}\,'(t) = \left\langle 1,\ \frac{2}{1+t^2},\ 3 + 2te^{t^2} \right\rangle$$

$$\vec{r}\,''(t) = \left\langle 0,\ -\frac{2\,(2t)}{(1+t^2)^2},\ 2e^{t^2} + 4t^2e^{t^2} \right\rangle$$

$$\vec{r}\,'(0) = \langle 1, 2, 3\rangle \qquad \vec{r}\,''(0) = \langle 0, 0, 2\rangle$$

$$a_T = \frac{\vec{r}\,'(0)\cdot\vec{r}\,''(0)}{|\vec{r}\,'(0)|} = \frac{6}{\sqrt{1^2+2^2+3^2}} = \boxed{\frac{6}{\sqrt{14}}} \approx 1.604$$

$$= \frac{6}{14}\sqrt{14} = \frac{3}{7}\sqrt{14}$$

(b) Use implicit differentiation to find $\dfrac{\partial z}{\partial y}$ for

$$xe^{2z} + x = \ln(x) + 2y^2z + e$$

at $(x, y, z) = (1, 1, 1/2)$.

$$2xe^{2z}\frac{\partial z}{\partial y} = 4yz + 2y^2\frac{\partial z}{\partial y}$$

$$\Rightarrow \quad \frac{\partial z}{\partial y} = \frac{4yz}{2xe^{2z} - 2y^2}$$

AT $(1, 1, \tfrac{1}{2})$

$$\boxed{\frac{\partial z}{\partial y} = \frac{4(1)(\frac{1}{2})}{2(1)e^1 - 2(1)} = \frac{2}{2e-2}}$$

$$= \frac{1}{e-1} \approx 0.5819$$

2. (14 pts) **The two parts below are not related.**

(a) Find and classify all critical points of $f(x,y) = 3xy - \frac{1}{2}y^2 + 2x^3 + \frac{9}{2}x^2$. (Clearly show your work in using the 2nd derivative test).

$$f_x = 3y + 6x^2 + 9x \overset{?}{=} 0$$
$$f_y = 3x - y \overset{?}{=} 0 \Rightarrow y = 3x$$

$$9x + 6x^2 + 9x \overset{?}{=} 0$$
$$6x^2 + 18x \overset{?}{=} 0$$
$$6x(x+3) \overset{?}{=} 0$$

$x = 0 \downarrow y = 0$; $x = -3 \downarrow y = -9$

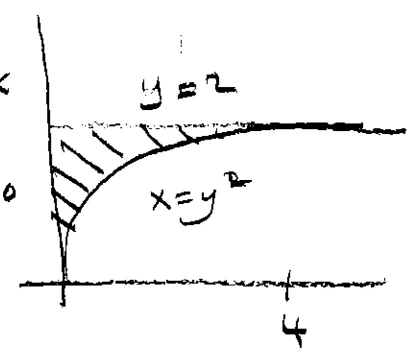
$$f_{xx} = 12x + 9, \quad f_{yy} = -1, \quad f_{xy} = 3$$

$\boxed{(0,0)} \Rightarrow f_{xx} = 9,\ f_{yy} = -1,\ f_{xy} = 3$; $D = -9 - (3)^2 = -18 < 0$ OR NOTICE DIFFERENT $\boxed{\text{SADDLE POINT}}$

$\boxed{(-3,-9)} \Rightarrow f_{xx} = -27,\ f_{yy} = -1,\ f_{xy} = 3$; $D = 27 - (3)^2 = 18 > 0$ CONCAVE DOWN IN ALL DIRECTIONS $\boxed{\text{LOCAL MAX}}$

(b) Set up and evaluate $\iint_D e^{y^3}\,dA$ where D is the region bounded by $y = \sqrt{x}$, $y = 2$ and $x = 0$.

$$\int_0^2 \int_0^{y^2} e^{y^3}\,dx\,dy \quad \text{OR} \quad \int_0^4 \Big(\underbrace{\int_{\sqrt{x}}^2 e^{y^3}\,dy}_{\text{CAN'T BE INTEGRATED!}} \Big) dx$$



$$\int_0^2 x e^{y^3} \Big|_0^{y^2} dy$$

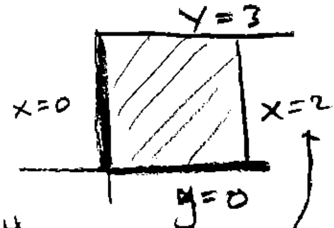
$$\int_0^2 y^2 e^{y^3}\,dy \qquad u = y^3,\ du = 3y^2\,dy,\ \tfrac{1}{3}du = y^2\,dy$$

$$= \int_0^8 \frac{1}{3} e^u\,du = \frac{1}{3} e^u \Big|_0^8$$

$$= \boxed{\frac{1}{3}(e^8 - 1)}$$

3. (14 pts) **The two parts below are not related.**

(a) Find the volume of the solid in the *first octant* bounded by the parabolic cylinder $z = 12 - 3x^2$ and the plane $y = 3$.



$\boxed{\text{STEP 1}}$ $z = 12 - 3x^2 \Rightarrow \iint_D 12 - 3x^2 \, dA$

$\boxed{\text{STEP 2}}$ $\boxed{A}$ DRAW $x=0, y=0, y=3$

$\boxed{B}$ DRAW INTERSECTION OF $z = 12 - 3x^2$ and $z = 0$ $\Big\}$ $x^2 = 4 \Rightarrow x = 2$

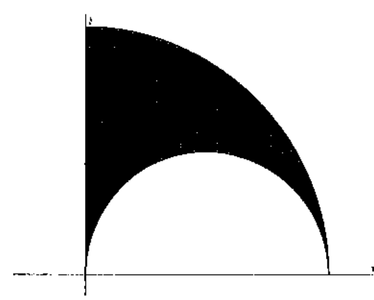
$$\int_0^3 \int_0^2 12 - 3x^2 \, dx \, dy$$

$$= \int_0^3 12x - x^3 \Big|_0^2 dy$$

$$= \int_0^3 24 - 8 \, dy = 16y \Big|_0^3 = \boxed{48}$$

(b) Set up and evaluate $\iint_D 3\sqrt{x^2 + y^2} dA$ where D is the region in the first quadrant that lies between the circles $\underbrace{x^2 + y^2 = 1}_{r = 1}$ and $\underbrace{x^2 + y^2 = x}_{r^2 = r\cos\theta}$.

$\Rightarrow r = \cos\theta$



$$\int_0^{\frac{\pi}{2}} \int_{\cos\theta}^1 3r \cdot r \, dr \, d\theta$$

$$\int_0^{\pi/2} r^3 \Big|_{\cos\theta}^1 d\theta$$

$$\int_0^{\pi/2} 1 - \cos^3\theta \, d\theta$$

$u = \sin\theta$
$du = \cos\theta \, d\theta$

$$\theta \Big|_0^{\pi/2} - \int_0^{\pi/2} (1 - \sin^2\theta) \cos\theta \, d\theta$$

$$\frac{\pi}{2} - \int_0^1 1 - u^2 \, du$$

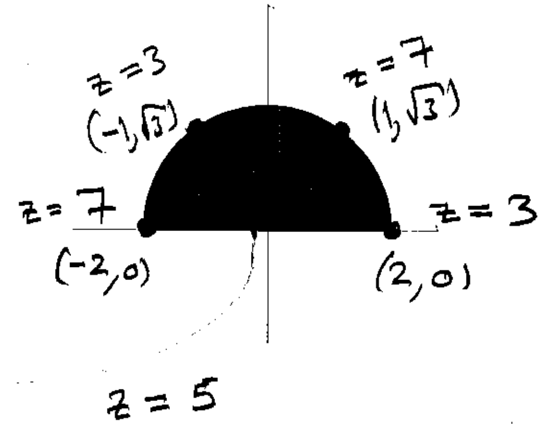
$$\frac{\pi}{2} - \left(u - \frac{1}{3}u^3 \Big|_0^1\right) = \frac{\pi}{2} - \left(1 - \frac{1}{3}\right) = \boxed{\frac{\pi}{2} - \frac{2}{3}}$$

4. (11 pts) Find the global maximum of $f(x,y) = xy^2 - x + 5$ over the region $y \geq 0$ and $x^2 + y^2 \leq 4$. There are two points where the global max occurs, also give these two points (show ALL your work including finding critical points inside the region and analyzing each boundary).

$\boxed{\text{I}}$ $f_x = y^2 - 1 \overset{?}{=} 0 \leftarrow y = \pm 1$

$f_y = 2xy \overset{?}{=} 0 \rightarrow x = 0$

$(0,1)$ & $(0,-1)$ NOT IN REGION



$\boxed{\text{II}}$ BOUNDARIES

$\boxed{A}$ $y = 0, \ -2 \leq x \leq 2 \Rightarrow z = f(x,0) = -x + 5$

LINEAR! MAX MUST OCCUR AT AN ENDPOINT

$z' = -1 \neq 0$ $\quad x = -2 \Rightarrow z = -(-2) + 5 = 7 \leftarrow$

NO CRITICAL NUMBERS! $\quad x = 2 \Rightarrow z = -(2) + 5 = 3$

$\boxed{B}$ $x^2 + y^2 = 4, \ -2 \leq x \leq 2 \Rightarrow z = x(4 - x^2) - x + 5$

$y^2 = 4 - x^2$ $\quad z = 4x - x^3 - x + 5$

$y = \sqrt{4 - x^2}$ $\quad z = 3x - x^3 + 5$

$z' = 3 - 3x^2 \overset{?}{=} 0 \Rightarrow x = \pm 1$

$x = 1 \Rightarrow y = \sqrt{4 - 1^2} = \sqrt{3} \Rightarrow z = 3 - 1 + 5 = 7$

$x = -1 \Rightarrow y = \sqrt{4 - (-1)^2} = \sqrt{3} \Rightarrow z = -3 + 1 + 5 = 3$

ALREADY CHECKED ENDPOINTS

ONLY THE FIVE POINTS SHOWN SHOULD BE CHECKED IF YOU UNDERSTAND THE PROCESS.

VERY IMPORTANT THAT YOU CHECKED THESE THREE THINGS.

+1

Global Max $= z =$ 7

Global Max occurs at $= (x,y) =$ $(-2,0)$ and $(1,\sqrt{3})$