

Math 126 - Fall 2018

Exam 1

October 23, 2018

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Student ID Number: \_\_\_\_\_

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- There are 4 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write  $\sqrt{4} = 2$  and  $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$  and  $\frac{7}{2} - \frac{3}{5} = \frac{29}{10}$  and  $\ln(1) = 0$  and  $\tan^{-1}(1) = \frac{\pi}{4}$ .
- Show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- If you need more room, use backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be **multiple versions** of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board.
- You have 50 minutes to complete the exam. Budget your time wisely.  
**SPEND NO MORE THAN 10 MINUTES PER PAGE!**

GOOD LUCK!

1. (13 pts)

(a) Multiple choice (circle **all** that apply, no work needed):

i. Let  $\ell$  be the line  $\mathbf{r}(t) = \langle -3t, 2t + 7, 6 - t \rangle$  and  $\mathcal{P}$  the plane  $6x - 4y + 2z = 4$ .  
Then  $\ell$  is...

(i) parallel to  $\mathcal{P}$

(ii) orthogonal to  $\mathcal{P}$

(iii) contained in  $\mathcal{P}$

(iv) intersects  $\mathcal{P}$  at one point

ii. Which of the following vector functions give points that are always on the curve of intersection of  $x^2 + z^2 = 4$  and  $x = 2y$ :

(i)  $\langle t, \frac{1}{2}t, \sqrt{4 - t^2} \rangle$

(ii)  $\langle 2 \cos(t), \cos(t), 2 \sin(t) \rangle$

(iii)  $\langle 2 \sin(t^3), \sin(t^3), 2 \cos(t^3) \rangle$

(iv)  $\langle 2t, t, 0 \rangle$

(b) A line,  $L$ , passes thru  $(1, 2, 2)$  and the *center* of the sphere,  $S$ , given by  $x^2 + y^2 + z^2 - 6z = 27$ .  
Find all  $(x, y, z)$  points of intersection of the line,  $L$ , and the sphere,  $S$ .

2. (12 pts)

- (a) Find an equation for the surface consisting of all points that are equidistant from the  $x$ -axis and the point  $(0, 0, 1)$  **AND** give a precise name for the corresponding 3D surface.

SURFACE NAME: \_\_\_\_\_

- (b) Find the equation of the plane that contains the points  $P(1, 5, 0)$  and  $Q(0, 8, 2)$  and is orthogonal to the plane  $x - 2y + z = 10$ .

3. (13 pts)

- (a) For some curve,  $\mathbf{r}(t)$ , you are given  $\mathbf{T}(0) = \left\langle \frac{3}{5}, 0, -\frac{4}{5} \right\rangle =$  ‘unit tangent at  $t = 0$ ’.  
(The three parts below are short answer and all refer to the vector  $\mathbf{T}(0)$  above)

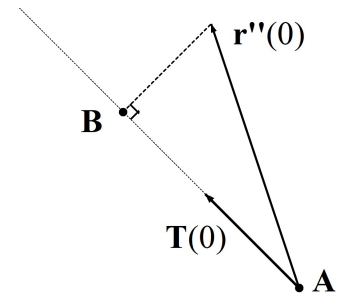
i. If  $|\mathbf{r}'(0)| = 3$ , then give the vector  $\mathbf{r}'(0) =$

- ii. If you are told that  $\mathbf{N}(0) = \langle 0, a, b \rangle =$  ‘the principal unit normal at  $t = 0$ ’ for some numbers  $a$  and  $b$ , then what can you conclude about  $a$  and  $b$ ? (list all possibilities)

$$a =$$

$$b =$$

- iii. If  $\mathbf{r}''(0) = \left\langle \frac{5}{4}, 2, -\frac{5}{3} \right\rangle$  is drawn tail-to-tail with  $\mathbf{T}(0)$  as shown below. What is the distance from point  $A$  to point  $B$  in the picture?



- (b) Find  $\mathbf{p}(t)$  if  $\mathbf{p}'(t) = \langle 3t^2, 2te^{t^2}, 4te^t \rangle$  and  $\mathbf{p}(0) = \langle 0, 0, 0 \rangle$ .

4. (12 pts)

(a) Find the angle of intersection (to the nearest degree) of the two curves

$$\mathbf{r}_1(t) = \langle t, 6 - 2t, 15 + t^2 \rangle \text{ and } \mathbf{r}_2(s) = \langle 5 - s, 2s - 4, s^2 \rangle.$$

(b) For  $x > 0$ , find the  $x$ -value at which curvature is maximum for  $f(x) = \frac{x^3}{3}$ .