MATH 126 – EXAM I Hints and Answers
Version Alpha
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1. HINT: Normal vectors of the given planes are $\vec{n}_1 = \langle 3, -2, 1\rangle$ and $\vec{n}_2 = \langle 2, 1, -3\rangle$. The direction vector for the line of intersection is orthogonal to both of these normal vectors. So, you may take as the direction vector for the line, the vector $\vec{v} = \vec{n}_1 \times \vec{n}_2 = \langle 5, 11, 7\rangle$.

   To find a point on the line, assume the line passes through the xy-plane and find the point $(x, y, 0)$ that is on both of the given planes; that is, set $z = 0$ in the equation of each plane and solve the resulting system of equations for x and y. The line of intersection pierces the xy-plane at the point $(1, 1, 0)$.

   ANSWER: Parametric equations for the line of intersection are:

   $$x = 1 + 5t,\ \ y = 1 + 11t,\ \ z = 7t.$$

   Other answers are possible.

2. HINT: The direction vector for the line is $\vec{a} = \langle -2, 3, 1\rangle$. A point on the line is $P_0(1, 0, -2)$. The vector from P_0 to the point $(3, 4, 3)$ is $\vec{b} = \langle 2, 4, 5\rangle$. A vector that is orthogonal to the plane is $\vec{n} = \vec{a} \times \vec{b} = \langle 11, 12, -14\rangle$. A vector that is orthogonal to the plane with length 10 is the vector $\dfrac{10\vec{n}}{|\vec{n}|}$.

   ANSWER: $\left\langle \dfrac{110}{\sqrt{461}}, \dfrac{120}{\sqrt{461}}, \dfrac{-140}{\sqrt{461}} \right\rangle$ (The vector $\left\langle \dfrac{-110}{\sqrt{461}}, \dfrac{-120}{\sqrt{461}}, \dfrac{140}{\sqrt{461}} \right\rangle$ is also acceptable.)

3. HINT: First, note that $\vec{r}(t) = \langle 3, 2, 0\rangle$ when $t = 1$ and that $\vec{r}\,'(t) = \langle \frac{1}{\sqrt{t}}, 3t^2 + 1, 3t^2 - 1\rangle$. Then ℓ is the line that passes through the point $(3, 2, 0)$ with direction vector $\vec{r}\,'(1) = \langle 1, 4, 2\rangle$. Parametric equations for ℓ are:

   $$x = 3 + t,\ \ y = 2 + 4t,\ \ z = 2t.$$

   To find where ℓ intersects the yz-plane, set $x = 3 + t$ equal to 0, solve for t and find the corresponding values of y and z.

   ANSWER: $(0, -10, -6)$

4. HINT: You can narrow down your choices for each function by evaluating each function at $\theta = 0$, which makes it clear that (a) and (b) should match the graphs on the right and (c) and (d) match those on the left. Then, you may want to consider how many solutions the equation $r = 0$ has for each function.

   ANSWER: top left: (d); top right: (a); bottom left: (c); bottom right: (b)

5. ANSWERS: $\vec{r}\,'(\pi) = \langle 0, 1, -3\rangle$; $\vec{r}\,''(\pi) = \langle 5, 0, 0\rangle$; $\kappa(\pi) = \frac{1}{2}$

6. HINT: $\cos^2 t = \frac{x}{3}$ and $\sin^2 t = -y$. So, $\frac{x}{3} - y = 1$. That is, the path of the particle lies on the line $y = \frac{x}{3} - 1$. The particle travels the portion of this path from the point $(3, 0)$ to the point $(0, -1)$ and back five times round-trip. The length of this portion of the path is $\sqrt{10}$.

   ANSWER: Distance traveled is $10\sqrt{10}$.

7. (a) ANSWER: $|z| = 2\sqrt{(x-2)^2 + (y+1)^2 + (z-3)^2}$

   (b) HINT: The above equation can be reduced to:

   $$1 = \frac{(x-2)^2}{3} + \frac{(y+1)^2}{3} + \frac{(z-4)^2}{4}.$$

   ANSWER: This is an ellipsoid.