

Math 125 Writing Up Problem 1: Some Sum Practice, then Mathematical Cakes
DUE FRIDAY, JANUARY 20

PROBLEM I: This is meant to be a quick exercise to give you a chance to play around with sigma notation and to preview an important sum that will re-appear in Math 126. For a given number a and a given positive integer n , the expression: $\sum_{i=0}^n a^i = 1 + a + a^2 + a^3 + \cdots + a^n$, is called a geometric sum.

Answer the following questions.

1. Multiply out and simplify the product: $(1 - a) \sum_{i=0}^n a^i = ???$.
2. Take your equality from the previous step and divide both sides by $1 - a$, to get the *shortcut* formula for the value of a geometric sum: $\sum_{i=0}^n a^i = \frac{???}{1 - a}$.
3. Use your formula to find the values of
 - (a) $1 + 3 + 3^2 + 3^3 + 3^4$.
 - (b) $1 + 3 + 3^2 + \cdots + 3^{100}$.
 - (c) $1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$.
 - (d) $1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots + \left(\frac{1}{2}\right)^{100}$.

4. Give the values of the limits (Do your answers make sense?):

(a) $\sum_{i=0}^{\infty} 3^i = \lim_{n \rightarrow \infty} \sum_{i=0}^n 3^i$.

(b) $\sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i = \lim_{n \rightarrow \infty} \sum_{i=0}^n \left(\frac{1}{2}\right)^i$.

PROBLEM II: Dr. Loveless likes to bake. For a departmental event, he is asked to make a cake in the shape a parabola. Viewed from above, the cake must look like the region bounded by the parabola $y = 1 - x^2$ and the x -axis (all units are in feet). Give nice, organized, and easy to follow answers to the following questions about this cake. Your answer should include labeled pictures.

1. The applied math department shows up and wants some cake. We say we'll give them half, but they must tell us where to make a **horizontal** cut in order to cut the cake precisely in half. Where should horizontal cut be?
2. The entire cake is served on a isosceles triangular tray. Viewed from above (on the same axes as the parabola), the tray has one side on the x -axis and the other two sides are tangent to the edge of the cake. If the tray is designed so that it's area is minimized, then what are the dimensions of the tray?
3. What are the areas of the visible parts of the tray (before the cake is cut)?