TEST PREP on 9.1, 9.3, and 9.4 - Dr. Loveless

These examples will come from the math department Math 125 Final Exam Archive. Look at the last two pages of almost any old exam and you will see more examples. If you are worried about "setting up problems", then do all the exercises on this practice sheet (solutions are included). Beyond that you'll get lots of practice in the homework with separating and solving. Make sure you understand how to manage your constants in a nice way. We'll be talking about that in class.

Winter 2016 - Final Exam - Problem 9 - Separate, integrate, manage constants

9. Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{x\cos(x^2)}{y}, \quad y(0) = -3.$$

Give your answer in the form $y = f(x)$.

Winter 2015 - Final Exam - Problem 9 - Separate, integrate, manage constants.

9. Find the solution of the initial value problem

$$\frac{dy}{dt} = (y + 2\sqrt{y})\cos^2(t), \quad y(0) = 1.$$

Give your answer in the form $y = f(t)$.

Winter 2013 - Final Exam - Problem 10 - Newton's Law of Cooling.

10. A 167°F cup of coffee is brought into a 59°F room. After two minutes, the temperature of the coffee is 149∘F. Assume the temperature of the room is constant, and that the rate of change of the temperature of the coffee is proportional to the difference between the temperature of the coffee and the temperature of the room. Let $T = T(t)$ denote the temperature (in °F) of the coffee as a function of time t (minutes).

    (a) Set up the differential equation and initial condition.

    (b) Solve the differential equation and find $y(t)$.

    (c) Find $T(4)$.

Winter 2015 - Final Exam - Problem 10 - A mixing problem.

10. At time $t = 0$, a 20-liter container filled with water has 400 gm of salt dissolved in it. Salt is being poured in at the top at the rate of 4 gm/min. In addition, water having a salt concentration of 3 gm/liter is entering the container at 2 liters/min, and the mixed water/salt solution leaves the container at the same rate of 2 liters/min. Assume that the salt is immediately well mixed in the water. Let $y = y(t)$ denote the amount of salt (in grams) in the container at time t (in minutes).

    (a) Set up the differential equation and initial condition.

    (b) Solve the differential equation and find $y(t)$.

    (c) Find $\lim_{t\to\infty} y(t)$

Spring 2013 - Final Exam - Problem 10 - A Savings Account (population problems are the same idea).

10. You would like to be a multimillonaire in 30 years. You might win the lottery, or you can start investing. This problem is about investing. Let $A(t)$ be the amount of money (in dollars) you have in your investment account at time t (in years). Let M be the amount (in dollars) that you deposit every month, so $12M$ is the amount (in dollars) that you deposit every year. The rate of change of the amount A in your account has two parts: the interest and your deposits. The part of the rate of change coming from the interest is proportional to the amount in your account, and the annual interest rate is 10and assume that your deposits are also applied continuously.

    (a) Set up the differential equation for $A(t)$. .

    (b) If $A(0) = 0$, then solve the differential equation and find $A(t)$.

    (c) Determine the value of M so that you will have a balance of exactly 5 million dollars after 30 years.