Sum of Consecutive Powers

The following story and challenge problems are optional. You are not required to turn this in.

Johann Carl Friedrick Gauss (1777 - 1855) is one of the most well-known mathematicians of all time. The following story is common folklore in the mathematical community:

One day when Gauss was in grade school, his teacher wanted to keep the class busy for awhile (or perhaps she was disciplining them). She asked that everyone add up all the whole numbers from 1 to 100. Remember that these students did not have calculators, so they had to carefully and tediously add up all the numbers. The story goes that Gauss solved the problem in a few seconds and show it to the teacher.

Let's see how you do on this problem. If you can handle the first problem, see if you can get through the second, third and forth.

1. Compute $1 + 2 + 3 + \cdots + 99 + 100 =$ ____________________

2. Find a formula for

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \underline{\hspace{6cm}}$$

3. The previous formula can be found in your textbook. Your textbook also has the following two formulas:

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

   See if you can find a formula for the forth powers. That is,

$$\sum_{i=1}^{n} i^4 = 1^4 + 2^4 + \cdots + n^4 = \underline{\hspace{6cm}}$$

   (the solution can be found on the web, but you should see if you can discover the pattern on your own. This is a difficult problem!)

4. (A very hard problem) See if you can find a general pattern for

$$\sum_{i=1}^{n} i^k = 1^k + 2^k + \cdots + n^k = \underline{\hspace{6cm}}$$