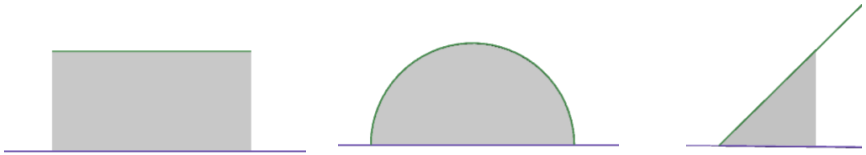
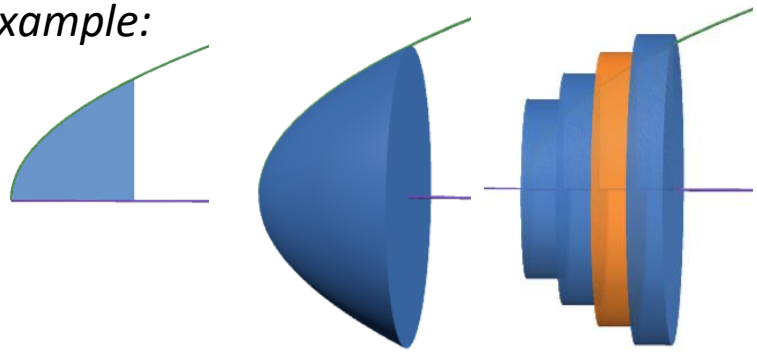
6.2 Volumes Using Cross-Sectional Slicing

Consider solids for which we can find a formula for the cross-sectional area.

Entry Task: What solids do you get when you rotate the following regions around the x-axis?

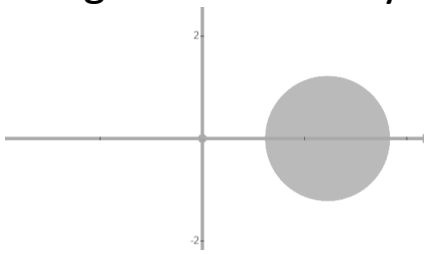


Another Example:



https://www.desmos.com/3d/ejury8aswi

What about this region about the y-axis…



Main Concept:

Find the general formula, $A(x_i)$, for the area of a cross-sectional slice at x_i then…

$$\text{Volume of one slice} \approx A(x_i)\Delta x$$

$$\text{Total Volume} \approx \sum_{i=1}^{n} A(x_i)\Delta x$$

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\Delta x$$

https://www.desmos.com/3d/3f1jcxyahq

https://www.desmos.com/3d/lypwxsa0im

Volume using cross-sectional slicing

1. Draw region.

   Cut **perpendicular** to rotation axis.

   Label *x* if cut crosses *x*-axis.

   Label *y* if cut crosses y-axis.

   Label *everything* in terms this variable.

2. Formula for cross-sectional area?

   disc: Area = $\pi(\text{radius})^2$

   washer: Area = $\pi(\text{outer})^2 - \pi(\text{inner})^2$

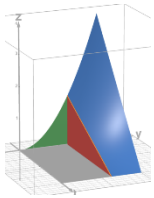
   square: Area = (Height)(Length)

   triangle: Area = ½ (Height)(Length)

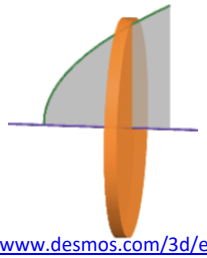
   Fill in using your labels.

3. Integrate the area formula.

$$Volume = \int_a^b A(x)dx$$

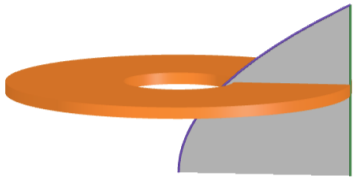


Example 1: Consider the region, *R*, bounded by $y = \sqrt{x}$, y = 0, and x = 1.
Find the volume of the solid obtained by rotating R about the **x-axis**.



https://www.desmos.com/3d/ejury8aswi

Example 2: Consider the region, R, bounded by $y = \sqrt{x}$, y = 0, and x = 1.
Find the volume of the solid obtained by rotating R about the ***y*-axis**.



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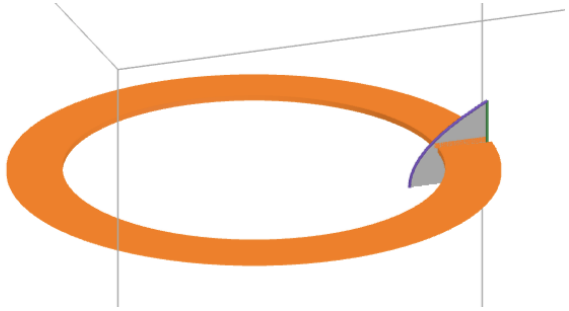
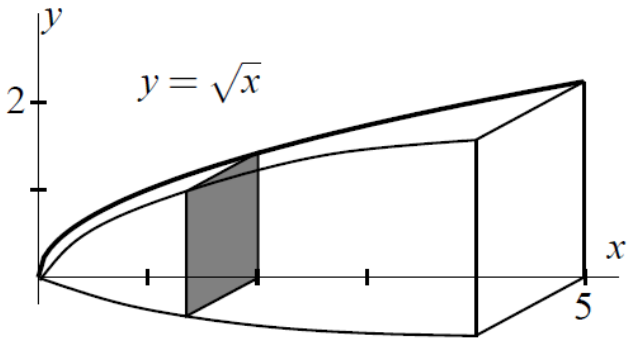
Example 3: Consider the region, R, bounded by $y = \sqrt{x}$, y = 0, and x = 1.
Find the volume of the solid obtained by rotating R about the vertical line x = -2.



Example 4: (an old final and HW)
Find the volume of the solid shown.
The cross-sections are squares.

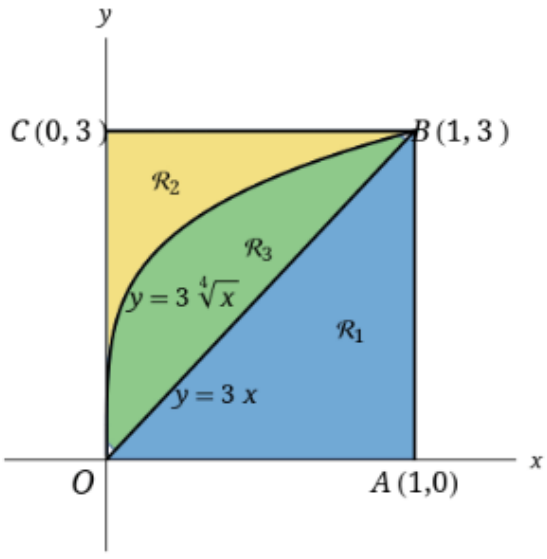


Example 5: *(from HW)*

Consider the region bounded by

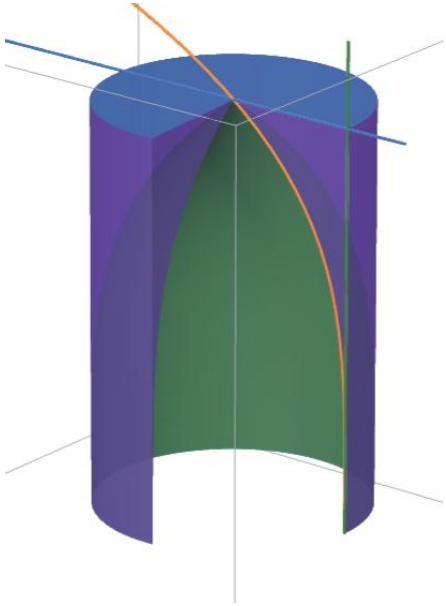
$$y = 3\sqrt[4]{x}\ , x = 0 \text{ and } y = 3.$$

Find volume of the solid obtained b rotating this region around the line x = 1.



Find the volume generated by rotating the given region about the specified line.

$\mathscr{R}_2$ about AB



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Summary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

This method has a major limitation:
6.2 method about *x-axis*, must use *dx*.
6.2 method about *y-axis*, must use *dy*.

What if the regions is rotated about the *x*-axis and we need to use *dy*?
(or about *y*-axis and we need dx?)

In these cases, 6.2 "Cross-sectional slicing" won't work!

We need another method. That is what we will do in 6.3.