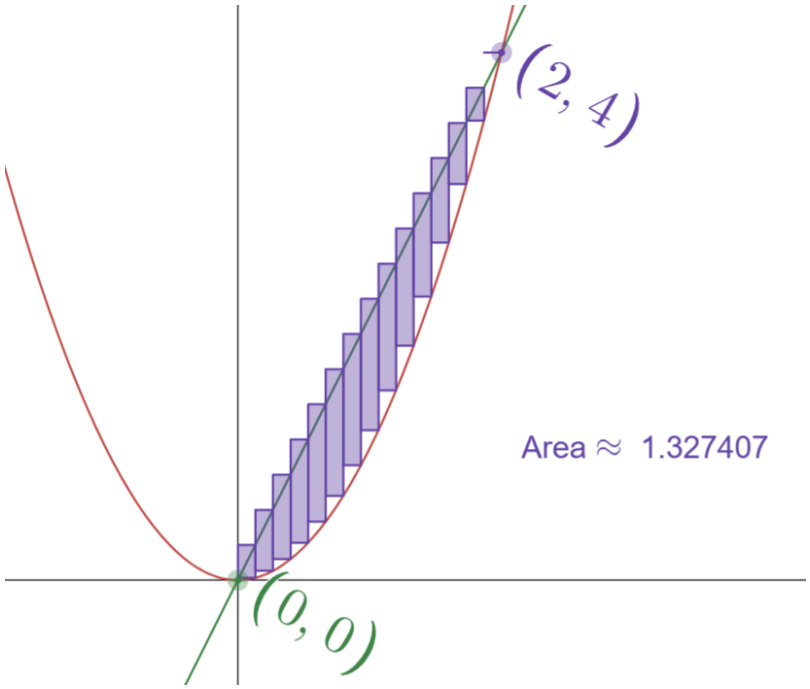
Ch 6: Basic Integral Applications

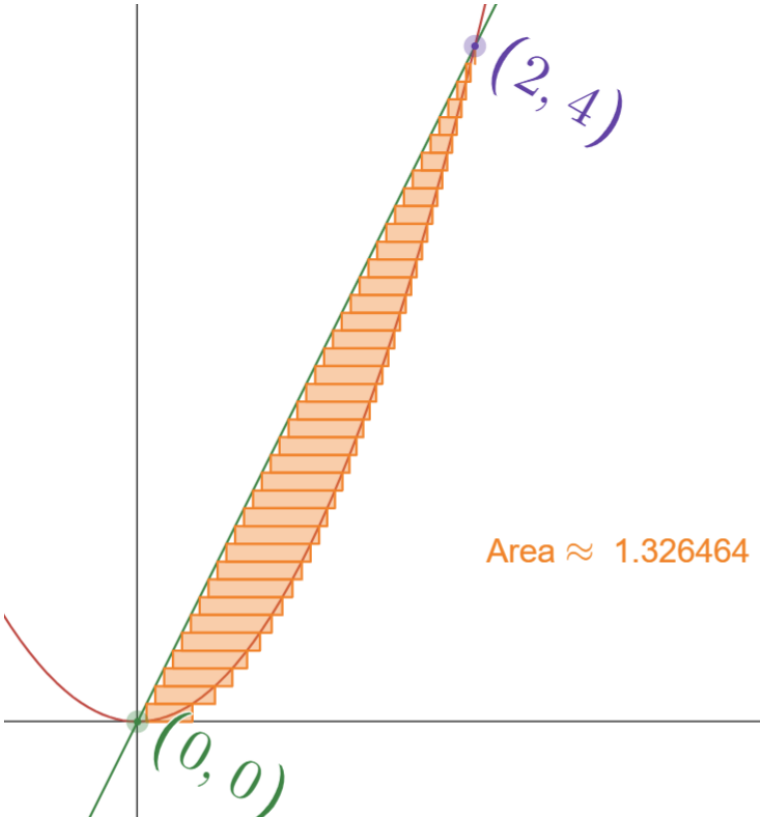
6.1 Areas Between Curves



https://www.desmos.com/calculator/aqnmxj1ozz

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i) - g(x_i))\Delta x$$

Example 1: Find the area bounded between $y = 2x$ and $y = x^2$.



https://www.desmos.com/calculator/aqnmxj1ozz

Example 1 (again): Find the area bounded between $y = 2x$ and $y = x^2$.

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} (f(y_i) - g(y_i))\Delta y$$

Summary: The area between curves

1. Draw picture, find intersections.
2. Choose dx or dy. Get ***everything*** in terms of the variable you choose.
3. Draw a typical approx. rectangle.
4. Set up as follows:

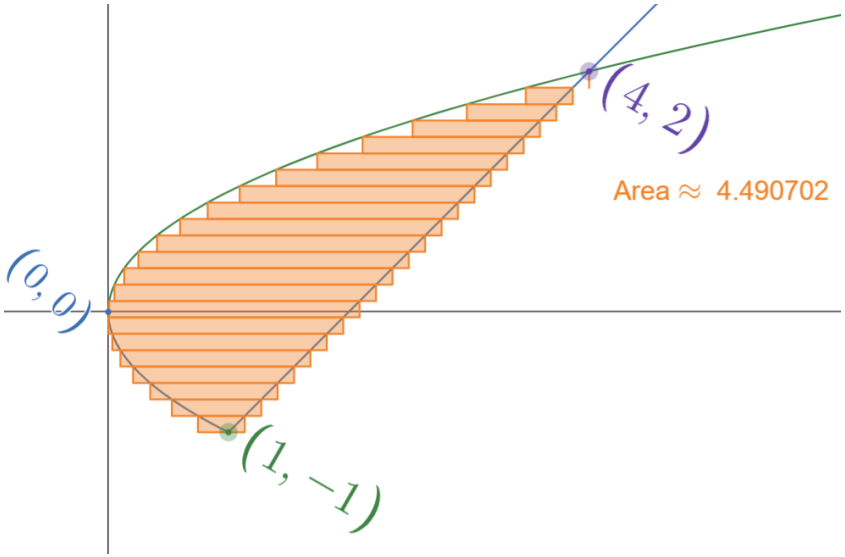
$$\text{Area} = \int_a^b (\text{TOP} - \text{BOTTOM})\, dx$$

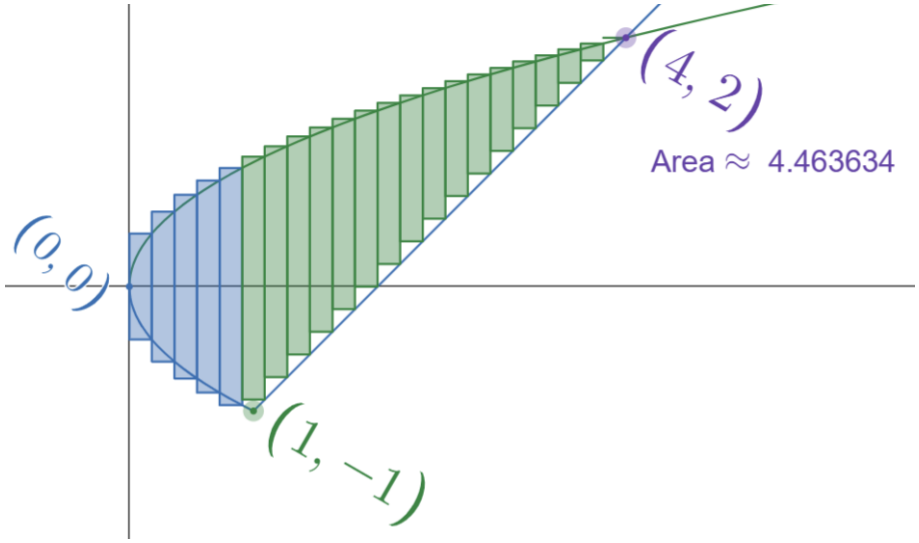
$$\text{Area} = \int_c^d (\text{RIGHT} - \text{LEFT})\, dy$$

Example 2:

Set up an integral (or integrals) that give the area of the region bounded by

$x = y^2$ and $y = x - 2$.


(4, 2)
Area ≈ 4.490702
(0, 0)
(1, −1)


(4, 2)
Area ≈ 4.463634
(0, 0)
(1, −1)

***"Fun"* examples to try on your own:**

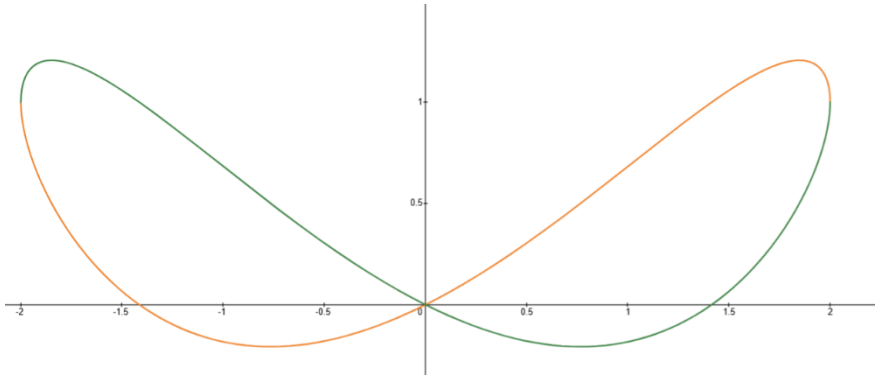
1. The gateway arch is approximately an inverted catenary given by the equation:

   $f(x) = 690 - 34\left(e^{(-0.01x)} + e^{(0.01x)}\right)$

   What is the area under the gateway arch?

2. Find the total area between the "satchel" curves given by

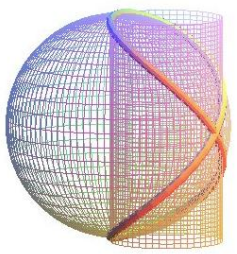
   $4y = x^2 + x\sqrt{4 - x^2}$ and $4y = x^2 - x\sqrt{4 - x^2}$



This is what the side view of the intersection of a cylinder and sphere looks like.

Viviani's curve, Vivianische Kurve

More info here (as well as a gallery of other curves)



Set up an integral for the total positive area of the following regions: