5.4 The Indefinite Integral and Net/Total Change

Net Change and Total Change

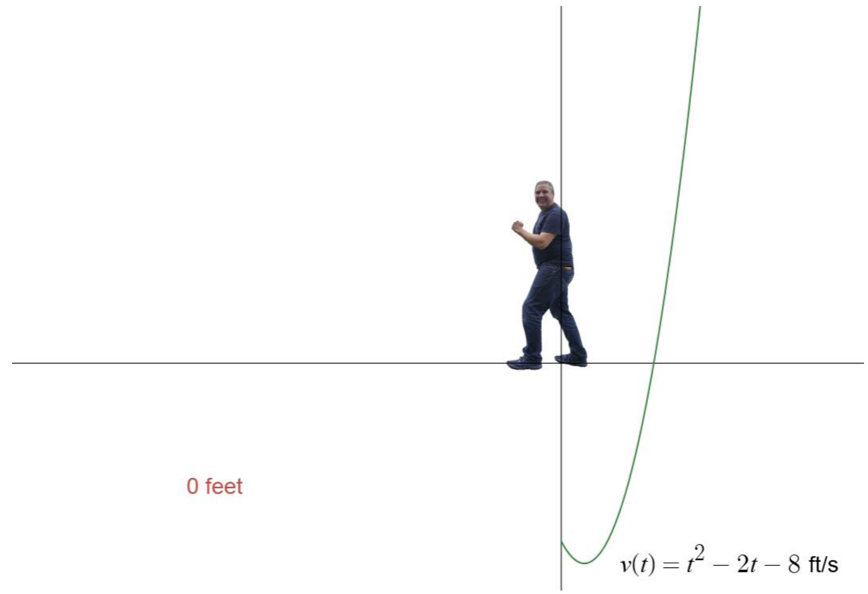
An object is moving on a straight line:

$s(t)$ = 'location at time *t*'

$v(t)$ = 'velocity at time t'

positive $v(t)$ means up/right

negative $v(t)$ means down/left



The FTOC (part 2) says

$$\int_a^b v(t)dt = s(b) - s(a)$$

We also call this the *displacement*.

Entry Task:

Let $v(t) = t^2 - 2t - 8$ ft/sec.

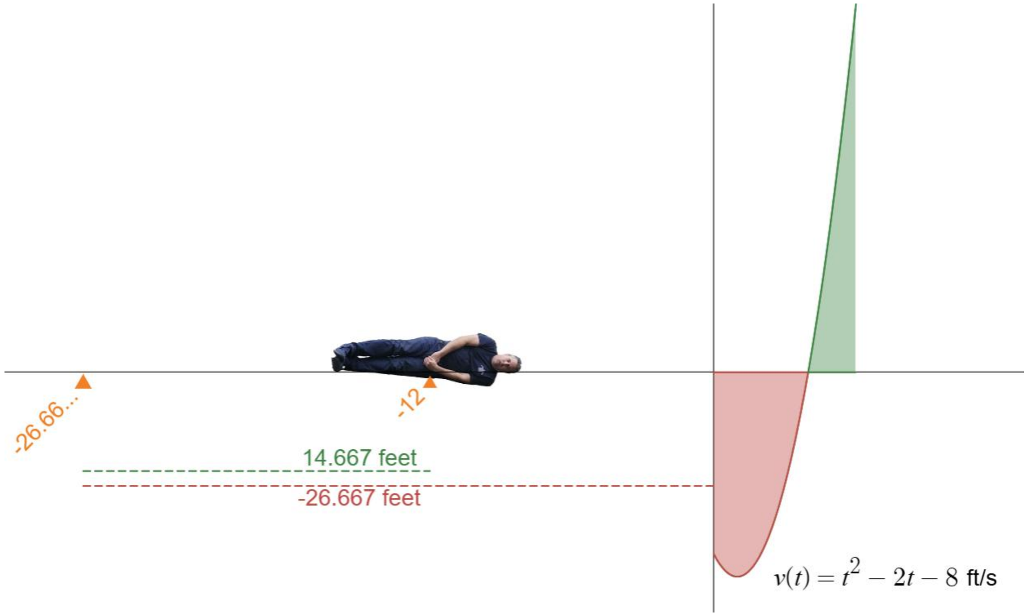
Compute displacement from t = 0 to t = 6.

We define **total change** in dist. by

$$\int_a^b |v(t)|dt$$

which we compute by

1. Solving $v(t) = 0$ for t.
2. Splitting up integral.
3. Adding together as positive numbers.



Visual: https://www.desmos.com/calculator/d42lbkvcfh

Example:

Let $v(t) = t^2 - 2t - 8$ ft/sec.

Compute the total distance traveled from t = 0 to t = 6.

A brief pause to discuss integration methods.

Examples (by simplifying we ***can*** do these):

1. $\int 6e^x + 4x - 5\sqrt{x}\,dx$
2. $\int \frac{\sqrt{x} - 3x}{x}\,dx$
3. $\int \frac{\cos(x)}{1 - \cos^2(x)}\,dx$

We ***cannot*** do now, but ***can later*** in term:

$\int xe^{3x}dx$ $\int \cos^3(x)dx$

$\int x\sin(x^2)\,dx$ $\int \frac{x^2 - x + 6}{x^3 + 3x}\,dx$

$\int \frac{3}{x - 2\sqrt{x}}\,dx$ $\int \frac{\sqrt{x^2 - 1}}{x^2}\,dx$

We will "***never***" do:

$\int e^{x^2}dx$ $\int \ln(x)\cos(x)\,dx$

$\int \frac{1}{x + e^x}\,dx$ $\int \sin(x^3)\,dx$

5.5 The Substitution Rule

Example 1 - Warm Up:

- What is the derivative of $y = (1 + x^4)^{11}$?

- Can you guess the integral…

$$\int x^3(1 + x^4)^{11}\, dx$$

The Substitution Rule:

If we write u = g(x) and du = g'(x) dx, then

$$\int f\big(g(x)\big)g'(x)dx = \int f(u)du$$

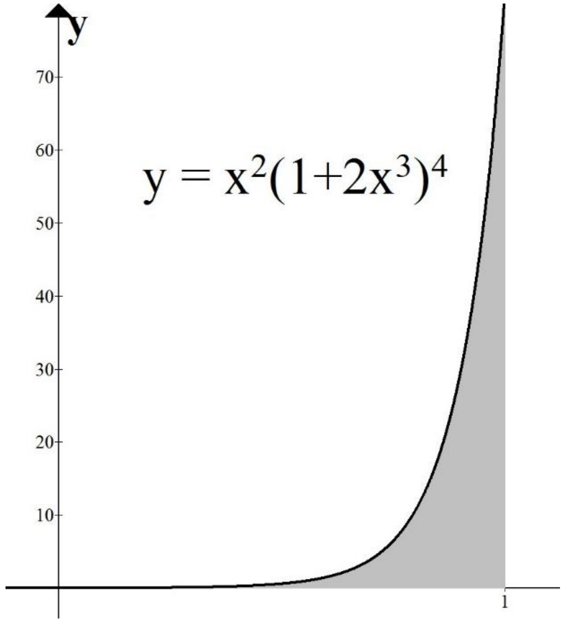
Observations:

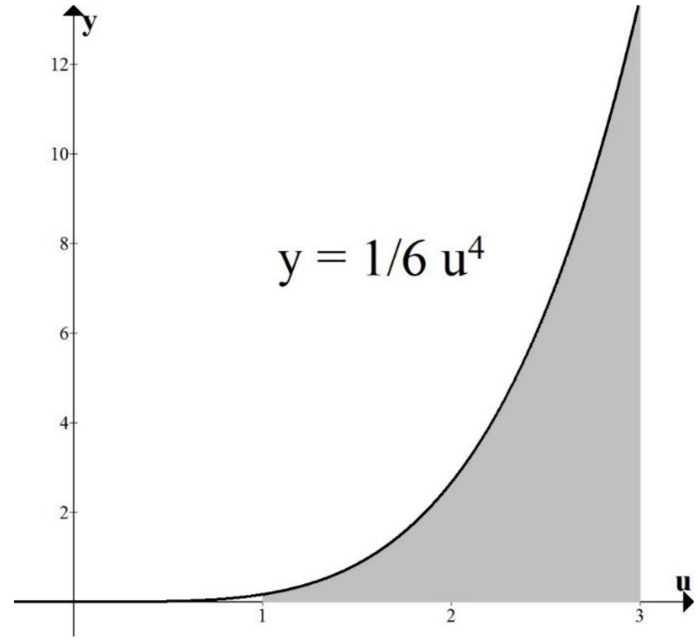
1. We are reversing the "chain rule".
2. We must see:

   "inside" = a function inside another

   "outside" = derivative of inside

Here is a visual:





$$\int_0^1 x^2(1+2x^3)^4 dx$$

$$\int_1^3 \frac{1}{6} u^4 du$$

Let $u = 1 + 2x^3$.
Change *everything* in terms of *u.*

Aside: What is really happening
(you do not need to write this)

Recall:

$$\int_a^b f\big(g(x)\big)g'(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\big(g(x_i)\big)g'(x_i)\Delta x$$

If we replace u = g(x), then we are "transforming" the problem from one involving *x* and *y* to one with *u* and *y*.

This changes ***everything*** in the set up.

The lower bound, upper bound, widths, and integrand all change!

Recall from Math 124 that

$$g'(x) = \frac{du}{dx} \approx \frac{\Delta u}{\Delta x}$$

(with more accuracy when Δx is small)

Thus, we can say that

$$g'(x)\Delta x \approx \Delta u$$

In other words, if the width of the rectangles using *x* and *y* is Δx, then the width of the rectangles using *u* and *y* is $g'(x)\Delta x$.

And if we write $u_i = g(x_i)$, then

$$\int_a^b f\big(g(x)\big)g'(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\big(g(x_i)\big)g'(x_i)\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f(u_i)\Delta u$$

$$= \int_{g(a)}^{g(b)} f(u)du$$

Example 2:

$$\int_{2}^{3} x^2 e^{x^3}\, dx$$

Advice on picking u = ?

Example 3: *Try u = inside*

$$\int \mathrm{x}\cos(\sin(x^2))\cos(x^2)\,dx$$

Example 4: *Try u = denominator*

$$\int_0^1 \frac{x}{x^2+3}\,dx$$

Example 5: *May need to simplify first*

$$\int \tan(x)\, dx$$

What to do when “old” variable remains

Example 6:

$$\int x^3\sqrt{2+x^2}\,dx$$

Example 7:

$$\int \frac{\mathrm{x}^7}{x^4+1}\,dx$$