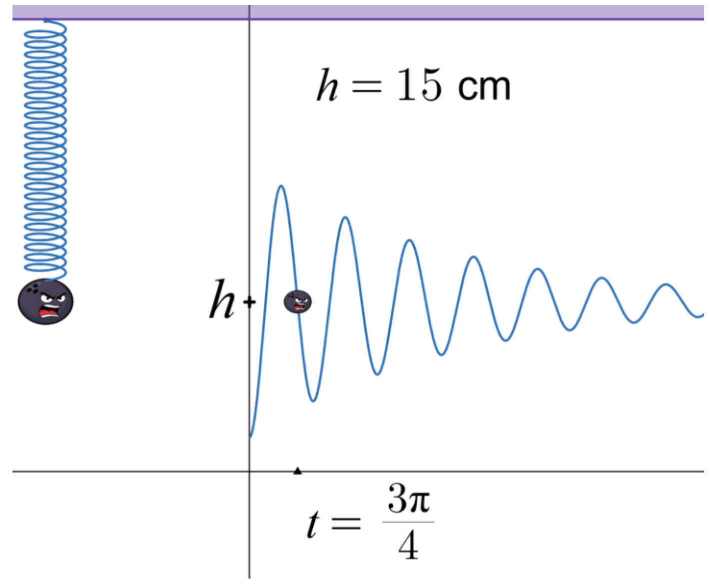
Entry Task: *Derivative Warm-Up*

Assume

$$h(t) = 15 - 12e^{-0.1t}\cos(2t)$$

gives the height in *cm* of a bowling ball on the end of an oscillating spring after *t* seconds.

What is the velocity at $t = \frac{3\pi}{4}$?



Animation: https://www.desmos.com/calculator/zsram9rvhb

4.9 Antiderivatives

Def'n: If g(x) = f'(x), then we say

g(x) = "the derivative of f(x)", and

f(x) = "an antiderivative of g(x)"

Example:

Give an antiderivative of

$$g(x) = x^2.$$

Examples (you do):

Find the general antiderivative of

1. $f(x) = x^6 + \sqrt{x}$
2. $g(x) = \cos(x) + \frac{1}{x} + e^x + \frac{1}{1+x^2}$
3. $h(x) = \frac{x-3x^2}{x^3}$

Initial Conditions

There is no way to know what “C” is without additional information. Such information is called an **initial condition**.

Example: $f'(x) = e^x + 4x$ and
$f(0) = 5$

Find $f(x)$.

Example: $f''(x) = 15\sqrt{x}$, and

$f(1) = 0, f(4) = 1$

Find $f(x)$.

Example: Assume I stand next to statue and throw a ball straight up. Assume that gravity provides a constant downward acceleration of 32 ft/s^2.

Let *f(t)* denote the height of the ball (in feet) above my hand at t seconds after it is thrown.

The ball leaves my hand at $t = 0$, reaches the top of the statue, and returns to my hand at $t = 3$ seconds.

Use calculus to determine the height the statue above my hand.

Video of me measuring statue heights:

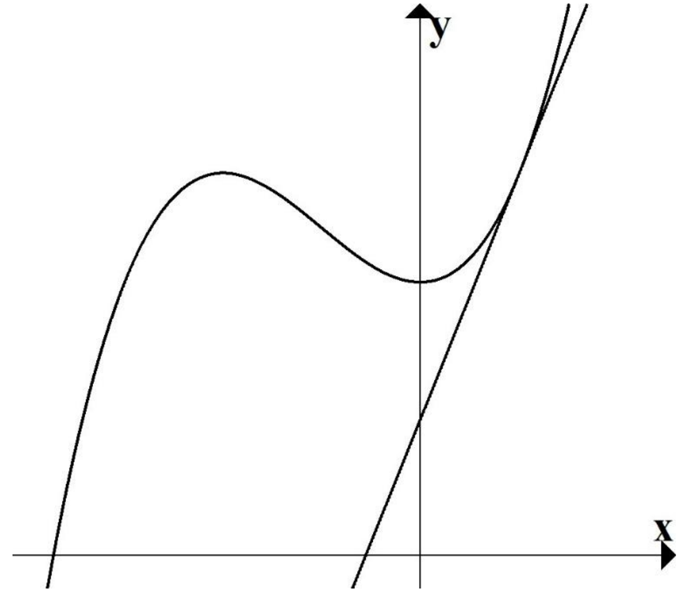
https://www.youtube.com/watch?v=07TDV5SgJT4

Preview of next class

5.1 Defining Area

Calculus is based on limiting processes "approaching" the
exact answer to a rate question.

Calc. I Visual:

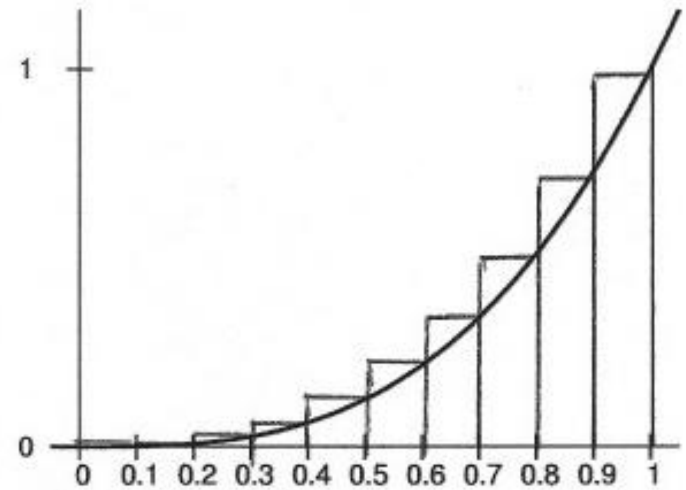


In Calculus I, you defined

f'(x) = `slope of the tangent at *x*'

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Calc. II Visual:



$R_{10} = 0.3025$

In Calculus II, we will see that antiderivatives are related
to the area `under' a graph

$$= \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i{}^*)\Delta x$$

Riemann sums set up:

A procedure to get approximations of the area “under” $f(x)$.

1. Break into *n* subintervals.

   $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

2. Draw *n* rectangles.

   Area of each rectangle =

   (height)(width) $= f(x_i^*)\Delta x$

3. Add up rectangle areas.