1. (13 pts) Evaluate the integrals. If you do a substitution in a definite integral problem, you must show me that you can appropriately change the bounds to get full credit. **Simplify your final answers.**

(a) $\int \frac{5}{e^{2x}} + 13 - \frac{\sqrt{9x^5}}{7\sqrt{x}}\,dx$ $= \int 5e^{-2x} + 13 - \frac{3}{7}\frac{x^{5/2}}{x^{1/2}}\,dx$

$= \int 5e^{-2x} + 13 - \frac{3}{7}x^2\,dx$ $\quad x \geq 0$

$= \boxed{-\frac{5}{2}e^{-2x} + 13x - \frac{1}{7}x^3 + C}$

(b) $\int \frac{x^2\sec^2(x^3)}{\tan^5(x^3)}\,dx$

$u = \tan(x^3)$

$du = \sec^2(x^3)\cdot 3x^2\,dx$

$\frac{1}{3}du = x^2\sec^2(x^3)\,dx$

$= \frac{1}{3}\int \frac{1}{u^5}\,du$

$= \frac{1}{3}\int u^{-5}\,du$

$= -\frac{1}{12}u^{-4} + C = \boxed{-\frac{1}{12}\tan^{-4}(x^3) + C}$

$= \boxed{-\frac{1}{12\tan^4(x^3)} + C}$

(c) $\int_1^{e^8} \frac{\sqrt[3]{\ln(x)}}{x}\,dx$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \int_0^8 u^{1/3}\,du$

$= \frac{3}{4}u^{4/3}\Big|_0^8 = \frac{3}{4}8^{4/3} = \frac{3}{4}16 = \boxed{12}$

2. (10 pts) (The two problems below are NOT related). **Simplify your final answers.**

(a) Evaluate $\int \frac{x}{(4+2x)^2}\,dx$

$u = 4+2x \rightarrow x = \frac{u-4}{2}$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$= \int \frac{1}{2}\frac{(u-4)}{u^2}\frac{1}{2}\,du$

$= \frac{1}{4}\int \frac{1}{u} - \frac{4}{u^2}\,du$ $\quad 4u^{-2}$

$= \frac{1}{4}\left[\ln|u| + 4u^{-1}\right] + C$

$= \boxed{\frac{1}{4}\ln|4+2x| + \frac{1}{4+2x} + C}$

(b) Find the area of the region bounded by $x = 3y$ and $y = \sqrt{x}$.

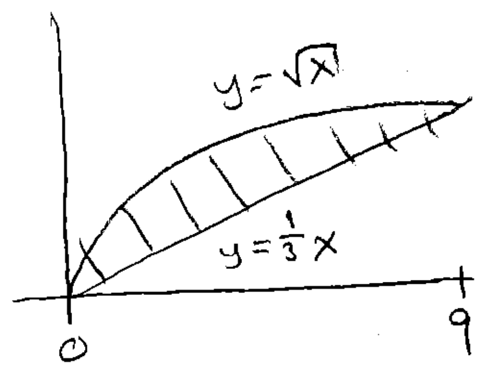
INTERSECTION: $x = 3\sqrt{x}$

$x^2 = 9x$

$x^2 - 9x = 0$

$x(x-9) = 0$

$x = 0,\ x = 9$



$\int_0^9 \sqrt{x} - \frac{1}{3}x\,dx$ OR $\int_0^3 3y - y^2\,dy$

$= \frac{2}{3}x^{3/2} - \frac{1}{6}x^2\Big|_0^9$

$= \frac{3}{2}y^2 - \frac{1}{3}y^3\Big|_0^3$

$= \frac{27}{2} - 9 = \frac{27}{2} - \frac{18}{2} = \frac{9}{2}$

$= \frac{2}{3}(9)^{3/2} - \frac{1}{6}(9)^2$

$= \frac{2}{3}27 - \frac{1}{6}81 = 18 - \frac{27}{2} = \frac{36}{2} - \frac{27}{2} = \boxed{\frac{9}{2} = 4.5}$

3. (13 pts) Leave your answers in exact form, but **simplify your final answers.**

(a) Consider $\lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\left(1+\frac{3i}{n}\right)^2}_{f(x_i)} \cdot \underbrace{\frac{3}{n}}_{\Delta x}$. Rewrite this as an integral and evaluate the integral.

$\frac{b-a}{n} = \frac{3}{n} \Rightarrow b-a=3$

$x_i = a + i\Delta x = 1 + \frac{3i}{n} \Rightarrow a = 1$

So $b = 4$

$\lim_{n\to\infty} \sum_{i=1}^{n} x_i^2 \Delta x = \int_1^4 x^2\,dx$

$= \frac{1}{3}x^3\Big|_1^4$

$= \frac{64}{3} - \frac{1}{3}$

$= \frac{63}{3} = \boxed{21}$

(b) Consider $g(x) = \int_0^x 2t + t\sin(\pi t^2)\,dt$.

i. Find $g'(1)$.

$g'(x) = 2x + x\sin(\pi x^2) \Rightarrow g'(1) = 2 + 1\sin(\pi) = \boxed{2}$ SLOPE

ii. Evaluate $g(1)$.

$\int_0^1 2t + t\sin(\pi t^2)dt = \int_0^1 2t\,dt + \int_0^1 t\sin(\pi t^2)dt$

$u = \pi t^2$

$du = 2\pi t\,dt$

$\frac{1}{2\pi}du = t\,dt$

$= t^2\Big|_0^1 + \int_0^{\pi} \frac{1}{2\pi}\sin(u)du$

$= 1 + \frac{1}{2\pi}\left[-\cos(u)\Big|_0^{\pi}\right]$

$= 1 + \frac{1}{2\pi}\left[(-(-1)) - (-1)\right] = 1 + \frac{2}{2\pi} = \boxed{1 + \frac{1}{\pi}}$ HEIGHT

iii. Give the equation for the tangent line to $g(x)$ at $x = 1$.
(Write your answer in the form $y = mx + b$)

$y = 2(x-1) + 1 + \frac{1}{\pi}$

$y = 2x - 1 + \frac{1}{\pi}$

SAME

4. (12 pts) (The two problems below are NOT related).

(a) Find $f(x)$, if $f''(x) = 28\sqrt[3]{x} - 6x$, $f(0) = 5$ and $f(1) = 10$. Put a box around your answer.

$$f''(x) = 28\,x^{1/3} - 6x$$

$$f'(x) = 21\,x^{4/3} - 3x^2 + C$$

$$f(x) = 9\,x^{7/3} - x^3 + Cx + D$$

$$f(0) = 5 \Rightarrow D = 5$$

$$f(1) = 10 \Rightarrow 9 - 1 + C(1) + 5 \stackrel{?}{=} 10$$

$$C + 13 = 10$$

$$C = -3$$

$$\boxed{f(x) = 9x^{7/3} - x^3 - 3x + 5}$$

(b) Compute $\displaystyle\int_1^8 \left|1 - \frac{16}{x^2}\right| dx$

$$1 - \frac{16}{x^2} \stackrel{?}{=} 0 \Rightarrow x^2 - 16 = 0 \Rightarrow x = \pm 4$$

$$\int_1^4 1 - 16x^{-2}\,dx = x + \frac{16}{x}\Big|_1^4 = \left(4 + \frac{16}{4}\right) - (1 + 16) = -9$$

$$\int_4^8 1 - 16x^{-2}\,dx = x + \frac{16}{x}\Big|_4^8 = \left(8 + \frac{16}{8}\right) - \left(4 + \frac{16}{4}\right) = 2$$

$$\text{TOTAL} = \int_1^8 \left|1 - \frac{16}{x^2}\right| dx = 9 + 2 = \boxed{11}$$

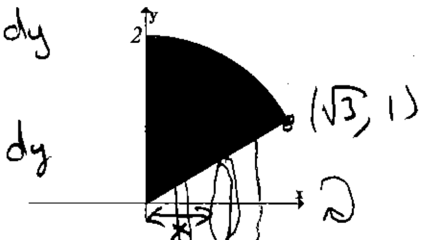
5. (12 pts) Consider the region, R, in the first quadrant that is bounded by the y-axis, the circle $x^2 + y^2 = 4$, and the line $\sqrt{3}y = x$ (shown below). You are given the picture multiple times for ease of labeling. Use any correct method.

(a) Set up (but DO NOT EVALUATE) an integral for the volume of the solid obtained by rotating R about the x-axis.

$$\boxed{\int_0^{\sqrt{3}} \pi\left(\sqrt{4-x^2}\right)^2 - \pi\left(\frac{x}{\sqrt{3}}\right)^2 dx}$$

OR

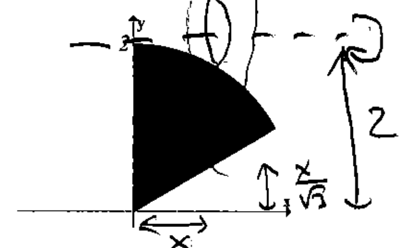
$$\int_0^1 2\pi y\cdot\sqrt{3}y\,dy + \int_1^2 2\pi y\sqrt{4-y^2}\,dy$$



(b) Set up (but DO NOT EVALUATE) an integral for the volume of the solid obtained by rotating R about the horizontal line $y = 2$.

$$\boxed{\int_0^{\sqrt{3}} \pi\left(2 - \frac{x}{\sqrt{3}}\right)^2 - \pi\left(2 - \sqrt{4-x^2}\right)^2 dx}$$

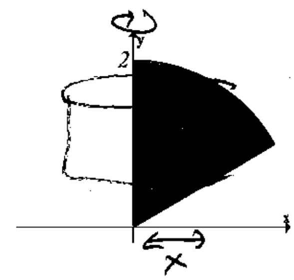
OR $\int_0^1 2\pi(2-y)\sqrt{3}y\,dy + \int_1^2 2\pi(2-y)\sqrt{4-y^2}\,dy$



(c) Find the volume of the solid obtained by rotating R about the y-axis. Hint: Shells! **Set-up AND evaluate.**

INTERSECTION: $(\sqrt{3}y)^2 + y^2 = 4 \Rightarrow 4y^2 = 4$

$\Rightarrow \boxed{\begin{array}{l} y = 1 \\ x = \sqrt{3} \end{array}}$



$$\int_0^{\sqrt{3}} 2\pi x\left(\sqrt{4-x^2} - \frac{1}{\sqrt{3}}x\right)dx$$

$$2\pi\left[\int_0^{\sqrt{3}} x\sqrt{4-x^2}\,dx - \underbrace{\int_0^{\sqrt{3}} \frac{1}{\sqrt{3}}x^2\,dx}\right]$$

$u = 4 - x^2$

$du = -2x\,dx$

$\frac{1}{-2}du = x\,dx$

$$= \frac{1}{3\sqrt{3}}x^3\Big|_0^{\sqrt{3}} = \frac{(\sqrt{3})^3}{3\sqrt{3}} = 1$$

$$2\pi\left[\int_4^1 \frac{-1}{2}u^{1/2}\,du - 1\right]$$

$$= 2\pi\left[-\frac{1}{3}u^{3/2}\Big|_4^1 - 1\right]$$

$$= 2\pi\left[\left(-\frac{1}{3} - -\frac{1}{3}\cdot 8\right) - 1\right] = 2\pi\left[\frac{7}{3} - 1\right] = 2\pi\cdot\frac{4}{3} = \boxed{\frac{8\pi}{3}}$$