1. (12 points) Compute:

(a) $\int \tan^3(5x)\sec^3(5x)\,dx$ $= \int \tan^2(5x)\sec^2(5x)\ \sec(5x)\tan(5x)dx$

$= \int (\sec^2(5x)-1)\sec^2(5x)\sec(5x)\tan(5x)dx$

$u = \sec(5x)$
$du = 5\sec(5x)\tan(5x)dx$

$= \frac{1}{5}\int (u^2-1)u^2\,du$

$= \frac{1}{5}\int u^4 - u^2\,du$

$= \frac{1}{5}\left(\frac{1}{5}u^5 - \frac{1}{3}u^3\right)+C$

$= \boxed{\frac{1}{25}\sec^5(5x) - \frac{1}{15}\sec^3(5x) + C}$

(b) $\int \sin^{-1}(x)\,dx$.

$u = \sin^{-1}(x)$ $\qquad dv = dx$
$du = \frac{1}{\sqrt{1-x^2}}dx$ $\qquad V = x$

$= x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}dx$

$t = 1-x^2$
$dt = -2x\,dx$
$-\frac{1}{2x}dt = dx$

$= x\sin^{-1}(x) - \int \frac{1}{-\sqrt{u}}\frac{1}{2}du$

$= x\sin^{-1}(x) + \frac{1}{2}\int u^{-1/2}du$

$= x\sin^{-1}(x) + \frac{1}{2}2u^{1/2} + C$

$= \boxed{x\sin^{-1}(x) + \sqrt{1-x^2} + C}$

2. (12 points) Compute:

(a) $\int_0^{\pi/2} \frac{\cos(x)\sin^2(x)}{\sin^2(x)+1}\,dx$

$t = \sin(x)$

$dt = \cos(x)dx$

$$= \int_0^1 \frac{u^2}{u^2+1}\,du$$

$$u^2+1 \overset{1}{\overline{\big)\, u^2}}$$

$$-(u^2+1)$$

$$-1$$

$$= \int_0^1 1 - \frac{1}{u^2+1}\,du$$

$$= u - \tan^{-1}(u)\Big|_0^1$$

$$= (1-\tan^{-1}(1)) - (0-\tan^{-1}(0))$$

$$= \boxed{1 - \pi/4} = \frac{4-\pi}{4} \approx 0.2146018\ldots$$

(b) $\int \sqrt{27 + 6x - x^2}\,dx$

$27 + 9 - 9 + 6x - x^2 = 36 - (x-3)^2$

$$= \int \underbrace{\sqrt{36-(x-3)^2}}\,\underset{\downarrow}{dx}$$

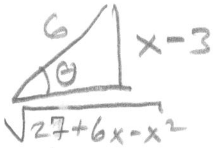
$x - 3 = 6\sin\theta$

$dx = 6\cos\theta\, d\theta$

$$= \int 6\cos\theta \quad 6\cos\theta\, d\theta$$

$\sin\theta = \frac{x-3}{6}$

$$= 36\int \tfrac{1}{2}(1+\cos(2\theta))\,d\theta$$



$$= 18\left(\theta + \underbrace{\tfrac{1}{2}\sin(2\theta)}\right) + C$$

$$= 18(\theta + \sin\theta\cos\theta) + C$$

$$= 18\sin^{-1}\left(\frac{x-3}{6}\right) + 18\,\frac{(x-3)}{6}\,\frac{\sqrt{27+6x-x^2}}{6} + C$$

$$= \boxed{18\sin^{-1}\left(\frac{x-3}{6}\right) + \frac{1}{2}(x-3)\sqrt{27+6x-x^2} + C}$$

↑

SAME AS $\sqrt{36-(x-3)^2}$

3. (12 points) Compute:

(a) $\int \frac{3x-2}{(x-1)(x+2)^2}\,dx$

$$\frac{3x-2}{(x-1)(x+2)^2} \overset{?}{=} \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$3x-2 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

- $x=1 \Rightarrow 1 = A \cdot 9 \Rightarrow A = 1/9$
- $x=-2 \Rightarrow -8 = C \cdot -3 \Rightarrow C = 8/3$
- COEF. OF x^2: $0 \overset{?}{=} A+B \Rightarrow B = -A = -1/9$

$$(x+2)^{-2} \longrightarrow \frac{1}{-1}(x+2)^{-1} + C$$

$$\int \frac{1/9}{x-1} - \frac{1/9}{x+2} + \frac{8/3}{(x+2)^2}\,dx$$

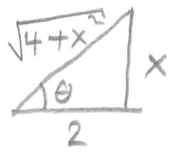
$$= \boxed{\frac{1}{9}\ln|x-1| - \frac{1}{9}\ln|x+2| - \frac{8/3}{x+2} + C}$$

$$= \frac{1}{9}\ln\left|\frac{x-1}{x+2}\right| - \frac{8}{3x+6} + C$$

(b) $\int \frac{1}{(4+x^2)^{3/2}}\,dx$

$$x = 2\tan\theta \xrightarrow{\tan\theta = x/2}$$

$$dx = 2\sec^2\theta$$



$$= \int \frac{1}{(4\sec^2\theta)^{3/2}}\, 2\sec^2\theta\, d\theta$$

$$= \int \frac{2\sec^2\theta}{8\sec^3\theta}\,d\theta = \frac{1}{4}\int \cos\theta\, d\theta$$

$$= \frac{1}{4}\sin\theta + C$$

$$= \boxed{\frac{1}{4}\frac{x}{\sqrt{4+x^2}} + C}$$

4. (12 points)

(a) Evaluate the improper integral or show that it diverges: $\int_{4/\pi}^{\infty} \frac{\cos(1/x)}{x^2}\, dx$

$$= \lim_{t\to\infty}\left[\int_{4/\pi}^{t} \frac{\cos(1/x)}{x^2}\, dx\right]$$

$u = \frac{1}{x} = x^{-1}$

$du = -x^{-2}dx$

$-x^2 du = dx$

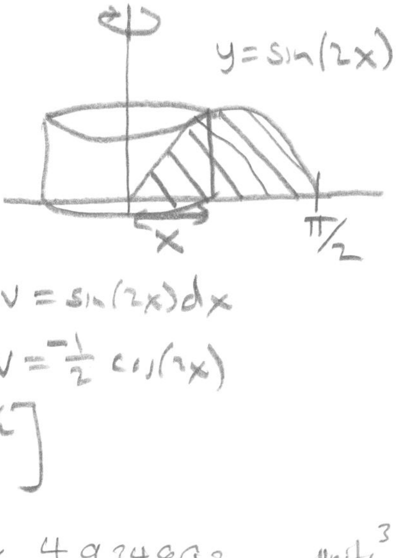
$$= \lim_{t\to\infty}\left[\int_{\pi/4}^{1/t} \frac{\cos(u)}{\cancel{x^2}} (-\cancel{x^2})du\right]$$

$$= \lim_{t\to\infty}\left[- \sin(u)\Big|_{\pi/4}^{1/t}\right]$$

$$= \lim_{t\to\infty}\left[-\sin(1/t) - -\sin(\pi/4)\right]$$

$$= -\sin(0) + \sqrt{2}/2 = \boxed{\sqrt{2}/2} \approx 0.70710678...$$

(b) Let R be the region bounded by $y = \sin(2x)$ and the x-axis and between $x = 0$ and $x = \pi/2$. Find the volume of the solid obtained by rotating this region about the y-axis. (Set up **and** evaluate)



$$\int_0^{\pi/2} 2\pi x \sin(2x) dx$$

$$= 2\pi\left[-\frac{1}{2}x\cos(2x)\Big|_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2}\cos(2x)dx\right]$$

$u = x$ $\quad dv = \sin(2x)dx$

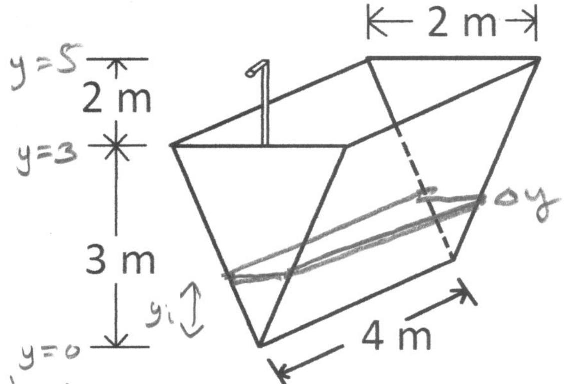
$du = dx$ $\quad v = \frac{-1}{2}\cos(2x)$

$$= 2\pi\left[\left(-\frac{1}{2}\left(\frac{\pi}{2}\right)\overbrace{\cos(\pi)}^{-1} - \overbrace{-\frac{1}{2}(0)\cos(0)}^{0}\right) + \frac{1}{4}\sin(2x)\Big|_0^{\pi/2}\right]$$

$$= 2\pi\left[\left(\frac{\pi}{4} - 0\right) + (0 - 0)\right] = \boxed{\frac{\pi^2}{2}} \approx 4.934802... \text{ units}^3$$

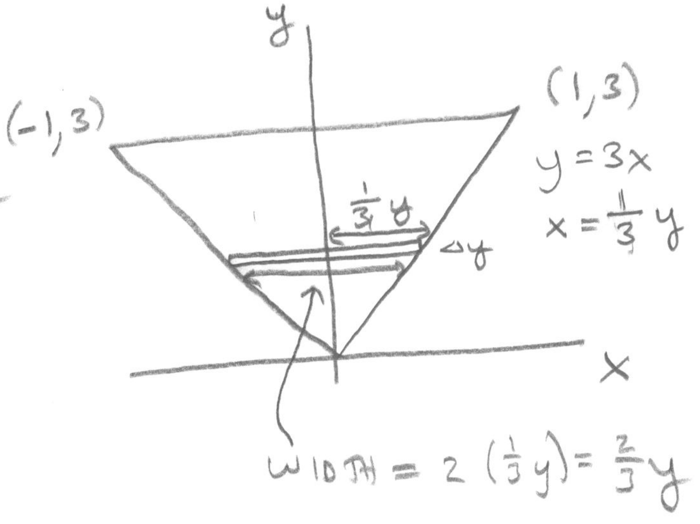
5. (12 points) A trough-shaped tank is full of water and all the water is going to be pumped up and out of a spout. The dimensions are shown below in meters. Note the top of the spout is 2 m above the top of the full tank (this is identical to a problem from homework, so interpret the picture in the same way as homework).
Find the work required to pump the water out of the spout shown. (include units)
Use 9.8 m/s^2 for the acceleration due to gravity and 1000 kg/m^3 for the density of water.

CONSIDER A HORIZONTAL SLICE OF WATER AT y_i WITH THICKNESS Δy



DIST. LIFTED $\approx 5 - y_i$ Meters

FORCE $\approx 9800 \cdot \underbrace{4 \cdot \frac{2}{3} y_i \, \Delta y}_{\text{Volume}}$ Newtons



$$\text{WORK} = \int_0^3 9800 \cdot 4 \cdot \tfrac{2}{3} y (5-y)\, dy$$

$$= 9800 \cdot \tfrac{8}{3} \int_0^3 5y - y^2 \, dy$$

$$= 9800 \cdot \tfrac{8}{3} \left[\tfrac{5}{2} y^2 - \tfrac{1}{3} y^3 \Big|_0^3 \right]$$

$$= 9800 \cdot \tfrac{8}{3} \underbrace{\left[\tfrac{45}{2} - 9 \right]}$$

$$= 9800 \cdot \tfrac{\overset{4}{8}}{3} \, \tfrac{\overset{9}{27}}{2} = 9800 \cdot 36 =$$

$$= \boxed{352{,}800 \text{ Joules}}$$