1. (11 pts) Evaluate the integrals. If you do a substitution, you *must* show that you know how to appropriately change bounds for full credit.

(a) $\displaystyle\int \frac{4}{\sqrt{x}} + x^4\left(\frac{3}{4x^5} - \frac{x^2}{2}\right) + \sec^2(5x)\, dx$

$$= \int 4x^{-1/2} + \frac{3}{4}\frac{1}{x} - \frac{1}{2}x^6\,dx + \int \sec^2(5x)\,dx$$

$$= 8x^{1/2} + \frac{3}{4}\ln|x| - \frac{1}{14}x^7 + \int \sec^2(u)\frac{1}{5}\,du$$

$u = 5x$
$du = 5\,dx$
$\frac{1}{5}du = dx$

$$\frac{1}{5}\tan(u) + C$$

$$= \boxed{8\sqrt{x} + \frac{3}{4}\ln|x| - \frac{1}{14}x^7 + \frac{1}{5}\tan(5x) + C}$$

(b) $\displaystyle\int_{\pi/3}^{\pi/2} \sin(x)e^{6\cos(x)}dx$

$u = 6\cos(x)$
$du = -6\sin(x)\,dx$
$\frac{1}{-6\sin(x)}du = dx$

$x = \pi/2 \Rightarrow u = 0$
$x = \pi/3 \Rightarrow u = 6\cdot\frac{1}{2} = 3$

$$= \int_3^0 \cancel{\sin(x)}\, e^u \frac{1}{-6\cancel{\sin(x)}}\,du$$

$$= -\frac{1}{6}\int_3^0 e^u\,du$$

$$= \frac{1}{6}\int_0^3 e^u\,du$$

$$= \frac{1}{6}\, e^u\Big|_0^3$$

$$= \boxed{\frac{1}{6}(e^3 - 1)}$$

2. (11 pts) Evaluate the integrals. If you do a substitution, you *must* show that you know how to appropriately change bounds for full credit.

(a) $\displaystyle\int \frac{e^{8x}}{(e^{4x}+5)^2}\,dx$

$u = e^{4x} + 5$ $\quad e^{4x} = u - 5$

$du = 4e^{4x}dx$

$\frac{1}{4e^{4x}} du = dx$

$$= \int \frac{e^{8x}}{u^2} \frac{1}{4e^{4x}} du$$

$$= \frac{1}{4} \int \frac{e^{4x}}{u^2} du$$

$$= \frac{1}{4} \int \frac{u-5}{u^2} du = \frac{1}{4} \int \frac{1}{u} - 5u^{-2} du$$

$$= \frac{1}{4} \left(\ln|u| + 5u^{-1} \right) + C$$

$$= \boxed{\frac{1}{4}\left(\ln(e^{4x}+5) + \frac{5}{e^{4x}+5}\right) + C}$$

(b) $\displaystyle\int_{-3}^{1} x\sqrt{x+3}\,dx$

$u = x + 3$ $\quad x = 1 \Rightarrow u = 4$

$du = dx$ $\quad x = -3 \Rightarrow u = 0$

$x = u - 3$

$$= \int_0^4 x \sqrt{u}\, du$$

$$= \int_0^4 (u-3)\, u^{1/2} du$$

$$= \int_0^4 u^{3/2} - 3u^{1/2} du$$

$$= \frac{2}{5} u^{5/2} - 2u^{3/2} \Big|_0^4$$

$$= \left(\frac{2}{5}(4)^{5/2} - 2(4)^{3/2} \right) - (0)$$

$$= \frac{2}{5} \cdot 32 - 2 \cdot 8 = \frac{64}{5} - 16 = \frac{64-80}{5} = \boxed{-\frac{16}{5}} = -3.2$$

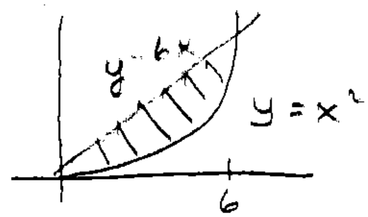
4. (12 pts)

(a) Consider the region bounded by $y = 6x$ and $y = x^2$ from $x = 0$ to $x = 6$.

i. Give the integral in terms of x for the area of this region. (DO NOT EVALUATE)

$$\boxed{\int_0^6 6x - x^2\,dx}$$

ACTUAL AREA $= 36$



ii. Approximate the area using $n = 3$ approximating rectangles and right endpoints (don't simplify, leave your answer expanded out).

$\Delta x = \frac{6-0}{3} = 2 \qquad x_0 = 0,\ x_1 = 2,\ x_2 = 4,\ x_3 = 6$

$$\boxed{(6(2) - (2)^2)\cdot 2 + (6(4) - (4)^2)\cdot 2 + (6(6) - (6)^2)\cdot 2} = 32$$

iii. Give the pattern, in terms of i and n, for the right endpoint approximation for this area with n subdivisions.

$\Delta x = \frac{6-0}{n}$

$x_i = \frac{6i}{n}$

Answer (enter pattern here): $\sum_{i=1}^{n} \boxed{\left(6\left(\frac{6i}{n}\right) - \left(\frac{6i}{n}\right)^2\right)\frac{6}{n}}$

(b) At time $t = 0$ seconds, you are standing on a bridge directly above Dr. Loveless and throw a small water balloon. It hits Dr. Loveless in 2 seconds with a downward speed of 75 feet/sec at the instant it hits him. Assume the balloon accelerates downward at a constant 32 feet/sec^2. At what height was the balloon exactly 1 second after you threw it?

$a(t) = -32$

$v(t) = -32t + C$

$h(t) = -16t^2 + Ct + D$

$v(2) = -75 \Rightarrow -32(2) + C = -75 \Rightarrow C = -75 + 64 = -11$

$h(2) = 0 \Rightarrow -16(2)^2 + C(2) + D = 0 \Rightarrow -64 - 22 + D = 0$

$\Rightarrow D = 86$

$h(t) = -16t^2 - 11t + 86$

$h(1) = -16(1)^2 - 11(1) + 86 = -27 + 86 = \boxed{59 \text{ feet}}$

3. (12 pts)

(a) Let $f(x) = \displaystyle\int_{5x+\cos(x)}^{32} \frac{4}{\sqrt{8+t}}\,dt$. Find $f'(0)$.

$$f'(x) = -\frac{4}{\sqrt{8+5x+\cos(x)}}\,(5-\sin(x)) \Rightarrow f'(0) = -\frac{4}{\sqrt{8+1}}\,(5-0)$$
$$= \boxed{-\frac{20}{3}}$$

(b) At t seconds, a particle is moving on a line with velocity $h'(t) = v(t) = 8t(t^2-1)^3$ ft/sec. At time $t = 2$ seconds, the particle is at a height of 10 ft (*i.e.* $h(2) = 10$).

i. Find the function, $h(t)$, for the height of the particle at time t seconds.

$$h(t) = \int 8t(t^2-1)^3\,dt \qquad u = t^2-1,\quad du = 2t\,dt,\quad \frac{1}{2t}\,du = dt$$
$$= \int 8\cancel{t}\,u^3\,\frac{1}{2\cancel{t}}\,du$$
$$= 4\,\frac{1}{4}u^4 + C$$
$$h(t) = (t^2-1)^4 + C$$
$$h(2) = 10 \Rightarrow (2^2-1)^4 + C = 10 \Rightarrow 81 + C = 10$$
$$C = -71$$
$$\boxed{h(t) = (t^2-1)^4 - 71}$$

ii. Find the **total distance** traveled by the particle from $t = 0$ to $t = 2$ seconds.

$$\int_0^2 |v(t)|\,dt = \int_0^2 \left|8t(t^2-1)^3\right|\,dt$$
$$8t(t^2-1)^3 \stackrel{?}{=} 0 \iff t=0,\ t=-1,\ \text{or}\ t=1$$
$$\int_0^1 8t(t^2-1)^3\,dt = (t^2-1)^4\Big|_0^1 = (0-(-1)^4) = -1$$
$$\int_1^2 8t(t^2-1)^3\,dt = (t^2-1)^4\Big|_1^2 = 3^4 - 0 = 81$$
$$\text{TOTAL DISTANCE} = 1 + 81 = \boxed{82\ \text{ft}}$$

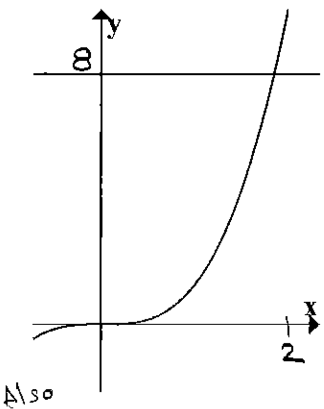
5. (12 pts) Let R be the region bounded by $y = x^3$, the **horizontal** line $y = 8$, and the y-axis.

(a) Find the value of the constant b such that the **horizontal** line $y = b$ would divide the region R into two regions of equal area.

$\leftarrow$ OR $\int_0^8 y^{1/3}\,dy$

TOTAL AREA $= \int_0^2 8 - x^3\,dx = 8x - \frac{1}{4}x^4 \Big|_0^2$

$= 16 - 4 = 12$



WANT $\int_0^b y^{1/3}\,dy \overset{?}{=} \frac{1}{2}(12)$

$\Rightarrow \frac{3}{4} y^{4/3}\Big|_0^b = \frac{3}{4} b^{4/3} \overset{?}{=} 6$

$\Rightarrow b^{4/3} = 8$

$\Rightarrow \boxed{b = 8^{3/4}} \approx 4.7568$

Also $\frac{8}{\sqrt[4]{8}} = \frac{8\sqrt[4]{2}}{\sqrt[4]{16}} = 4\sqrt[4]{2}$

(b) Find the volume of the solid obtained by rotating the region R about the y-axis. Set up AND evaluate. (Use any correct method)

$\int_0^8 \pi (y^{1/3})^2\,dy = \pi \frac{3}{5} y^{5/3}\Big|_0^8$ OR $\int_0^2 2\pi x\,(8 - x^3)\,dx$

$= \frac{3\pi}{5} 8^{5/3}$

$= \frac{3\pi}{5} \cdot 32$

$= \boxed{\frac{96\pi}{5}} \approx 60.319$

(c) A solid is obtained by rotating the region R around the **horizontal** line $y = -1$. Set up the integrals you get for the volume of this solid using BOTH the method of cylindrical shells and the method of washers (DO NOT EVALUATE).

Shells: $\boxed{\int_0^8 2\pi (y+1)\, y^{1/3}\,dy}$ ≈ 420.08

Washers: $\boxed{\int_0^2 \pi (8+1)^2 - \pi (x^3+1)^2\,dx}$