1 (14 pts) Determine the values of the following limits, or state that the limit does not exist. If it is correct to say that the limit is $+\infty$ or $-\infty$, then you should say so. Show your work.

a) $$\lim_{x\to\infty}\left(\frac{\sqrt{x^2-4x}}{3x-12}\right)=\lim_{x\to\infty}\left(\frac{\frac{1}{x}\sqrt{x^2-4x}}{\frac{1}{x}(3x-12)}\right)=\lim_{x\to\infty}\left(\frac{\sqrt{1-\frac{4}{x}}}{3-\frac{12}{x}}\right)=\frac{\sqrt{1-0}}{3-0}=\boxed{\frac{1}{3}}$$

b) $$\lim_{x\to\pi^-}(\ln(\sin(x))=\lim_{y\to 0^+}(\ln(y))=\boxed{-\infty}$$

c) $$\lim_{t\to 3}\left(\frac{\sqrt{t+6}-t}{t-3}\right)=\lim_{t\to 3}\left(\frac{\sqrt{t+6}-t}{t-3}\times\frac{\sqrt{t+6}+t}{\sqrt{t+6}+t}\right)=\lim_{t\to 3}\left(\frac{t+6-t^2}{(t-3)(\sqrt{t+6}+t)}\right)=$$

$$=\lim_{t\to 3}\left(\frac{-(t^2-t-6)}{(t-3)(\sqrt{t+6}+t)}\right)=\lim_{t\to 3}\left(\frac{-(t-3)(t+2)}{(t-3)(\sqrt{t+6}+t)}\right)=\lim_{t\to 3}\left(\frac{-(t+2)}{\sqrt{t+6}+t}\right)=\frac{-(3+2)}{\sqrt{3+6}+3}=\boxed{-\frac{5}{6}}$$

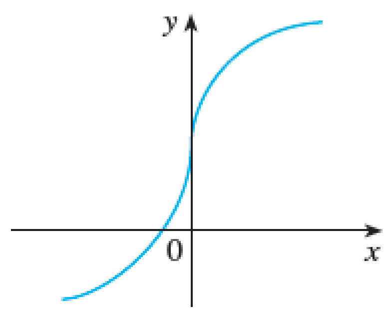
2 (6 pts) Compute the slope of the tangent line to $y=\frac{1}{\sqrt[3]{x}}+\frac{2x+7}{x}$ at the point (1, 10)

$$y=x^{-\frac{1}{3}}+2+7x^{-1}$$

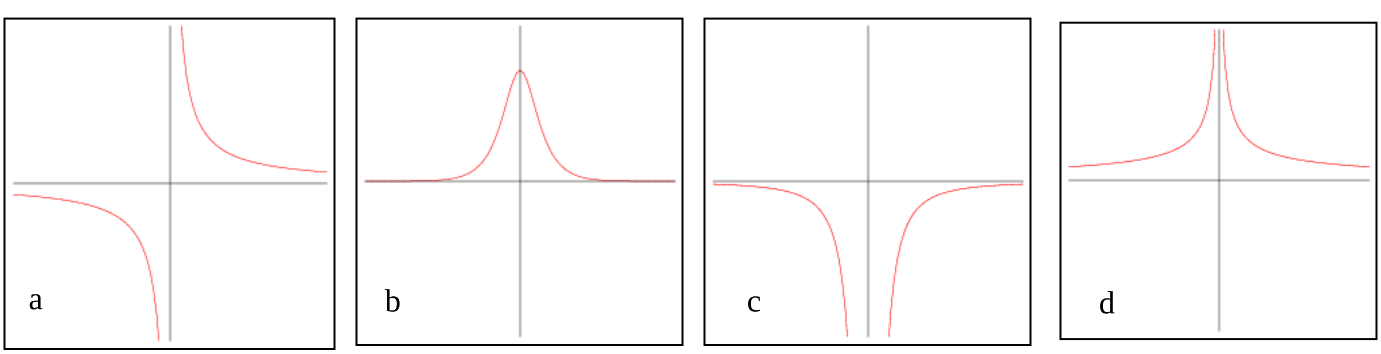
$$\frac{dy}{dx}=-\frac{1}{3}x^{-\frac{4}{3}}-7x^{-2}$$

$$slope=\left.\frac{dy}{dx}\right|_{x=1}=-\frac{1}{3}-7=\boxed{-\frac{22}{3}}$$

3 (4 pts) The graph on the right is the graph of a function f.



Which one of the following 4 graphs could be the graph of its derivative?



Graph of f' is :___(d)_____ (no need to justify)

4 (15 pts) Consider the function Consider the function:

$$f(x) = \begin{cases} 2x+3 & \text{if } x \le 0 \\ \dfrac{4}{x+1} & \text{if } 0 < x \le 1 \\ 2\sqrt{x} & \text{if } 1 < x \end{cases}$$

a) (4 pts) Compute the following four limits of this function:

$\lim_{x\to 0^-} f(x) = 3$ $\lim_{x\to 1^-} f(x) = 2$

$\lim_{x\to 0^+} f(x) = 4$ $\lim_{x\to 1^+} f(x) = 2$

(3 pts) List all the points where this function f is **discontinuous**. For each point of discontinuity, specify the type: removable, jump or infinite.

$x = 0$ (jump)

b) (5 pts) Compute the derivative of f. Write it in bracket notation as above, with correct domain for each part.

$$f'(x) = \begin{cases} 2 & \text{if } x < 0 \\ \dfrac{-4}{(x+1)^2} & \text{if } 0 < x < 1 \\ \dfrac{1}{\sqrt{x}} & \text{if } 1 < x \end{cases}$$

c) (3 pts) List all real numbers where f is **not differentiable** and justify why (discontinuity, corner, or vertical tangent)

Not differentiable at $x = 0$ (discontinuous) and $x = 1$ (corner)

5 (7 points) (7 points) An object moves in the xy-plane. Its coordinates at time t seconds are given by the parametric equations:

$$x(t) = t\cos(t)$$

$$y(t) = t\sin(t)$$

Both coordinates are measured in inches, and the time is measured in seconds.

a) Compute the horizontal velocity of this object at time 0 seconds.

(Recall that the horizontal velocity is the instant rate of change of the x-coordinate)

Include correct units in your answer.

$$x'(t) = 1\cos(t) + t(-\sin(t)) = \cos(t) - t\sin(t)$$

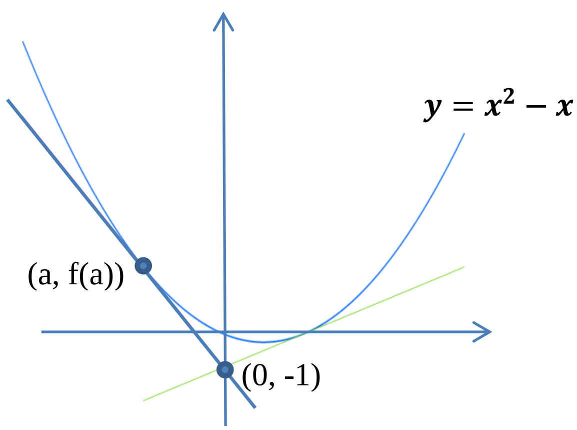
$$x'(0) = \boxed{1\ in/sec}$$

b) Write a formula in terms of t for the distance $d(t)$ between the origin (0,0) and the position of this object at t seconds. Simplify your formula.

$$d(t) = \sqrt{(x(t)-0)^2 + (y(t)-0)\text{^}2} = \sqrt{t^2\cos^2(t) + t^2\sin^2(t)} = \sqrt{t^2} \boxed{= |t|}$$

Recall: In general, it is not true that $\sqrt{x^2} = x$. Rather, $\boxed{\sqrt{x^2} = |x|}$ (which is x only if x is positive!)

$\boxed{6}$ (6 points) Determine the equation of the tangent line to the graph of $y = x^2 - x$, which passes through the point $(0, -1)$ and whose point of tangency P is in the second quadrant. See the picture below.



$y' = 2x - 1$

We don't know the coordinates of the point P of tangency, so we label it $(a, f(a))$.

We need to compute a.

There are a few different ways to get an equation in a. I'll use my favorite one, which is to write the slope of the tangent line two different ways: as the derivative at a, and as the rise over the run between $(a, f(a))$ & (0, -1):

$$2a - 1 = \frac{(a^2 - a) - (-1)}{a - 0}$$

$$2a - 1 = \frac{a^2 - a + 1}{a}$$

Solving, we get: $a = \pm 1$. The point P we want is $(a, f(a)) = (-1, 2)$

Tangent line at P has equation: $\boxed{y = -3x - 1}$