# Math 124 - Winter 2016
# Exam 1
# February 2, 2016

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

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- There are 6 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. In such an instance, you will meet in front of a board of professors to explain your actions.
  DO NOT CHEAT OR DO ANYTHING THAT LOOKS SUSPICIOUS!
  WE WILL REPORT YOU AND YOU MAY BE EXPELLED!
  Keep your eyes down and on your paper. If your TA sees your eyes wandering they will warn you only once before taking your exam from you.
- You have 80 minutes to complete the exam. Budget your time wisely.
  **SPEND NO MORE THAN 15 MINUTES PER PAGE!**

GOOD LUCK!

1. (9 pts) Determine the values of the following limits or state that the limit does not exist. If it is correct to say that the limit equals $\infty$ or $-\infty$, then you should do so. **In all cases, show your work/reasoning. You must use algebraic methods where available. And explain in words your reasoning if an algebraic method is not available.**

   (a) $\displaystyle \lim_{x\to -3} \frac{\frac{1}{x^2} - \frac{1}{9}}{x+3}$

   (b) $\displaystyle \lim_{x\to \pi} \left(5 + \frac{\cos(x)}{3\sin^2(x)}\right)$

   (c) $\displaystyle \lim_{x\to \infty} \frac{6x - \sqrt{9x^2 - 1}}{10 + 5x}$

2. (9 pts) Find the indicated derivatives. (Hint: Simplify **before** you start applying derivative rules).

(a) $y = (2x^2)^3 - \dfrac{4x^3}{6} + \dfrac{e^x}{2}$, find $\frac{dy}{dx}$.

(b) $v = \left(\sqrt{x} + \dfrac{1}{\sqrt[5]{x}}\right)^2$, find $v'$.

(c) $f(x) = \dfrac{5x}{x^4} + 8x^2 \csc(x)$, find $f'(x)$.

**Hint: Parts (b) and (c) can be done independently of part (a) and you can use them to check your work in part (a)!**

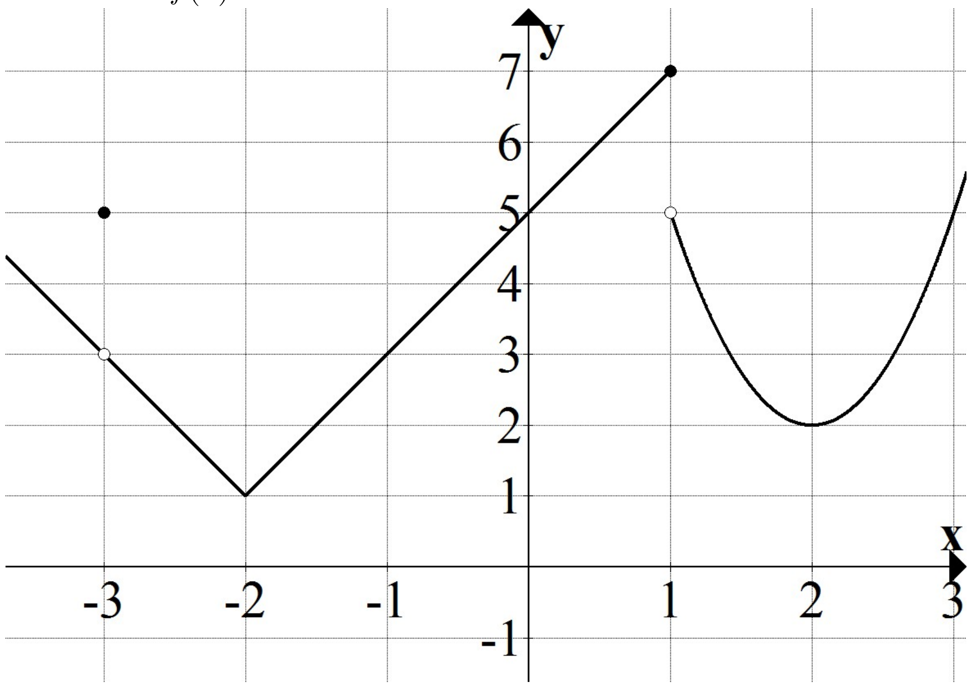
3. (9 pts) You are keeping track of your child's height. You roughly find that the height in centimeter (cm) of your child is given by $f(t) = 60 + 30\sqrt{t}$ where $t$ is the child's age in years.

   (a) (5 pts) Find and *completely simplify* the expression $\dfrac{f(t+h) - f(t)}{h}$.
   (Simplify until the $h$ in the denominator cancels)

   (b) (2 pts) Find the average rate of change of the height with respect from $t = 1$ to $t = 9$ years.
   (Include units in your answer).

   (c) (2 pts) Find the instantaneous rate of change when the child turns $t = 16$ years old.
   (Include units in your answer).

4. Consider the function $f(x)$ shown:




(a) (2 pts) Find all solutions to $f'(x) = 0$.

(b) (2 pts) Name all values of $x$ at which $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ does not exist.

(c) (2 pts) Find $\lim_{x \to -3} (6f(x) + f(x+3))$

(d) (3 pts) If $g(x) = x^3 f(x)$, then find $g'(-1)$.

5. (12 pts)

(a) Let $a$ be a constant and consider the function $f(x) = \begin{cases} \dfrac{2-\sqrt{x}}{4-x} & \text{, if } x < 4; \\ \cos\left(\frac{\pi}{4}x\right) + a\sqrt{x} & \text{, if } x \geq 4. \end{cases}$

Find the value of $a$ that will make $f(x)$ continuous for all values of $x$.

(b) Let $b$ be a constant and consider the function $g(x) = \begin{cases} bx + 10e^x & \text{, if } x < 0; \\ 10 + \sin(x) - 4\tan(x) & \text{, if } x \geq 0. \end{cases}$

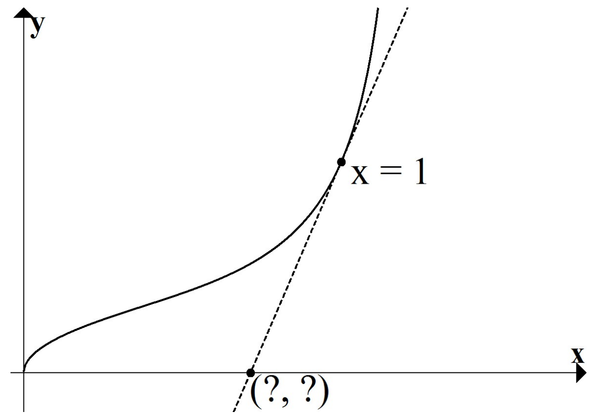
The function $g(x)$ is continuous at $x = 0$ (you don't have to show this).

Find the value of $b$ that makes the function differentiable at $x = 0$.

(Hint: Use your derivative shortcut rules!)

6. (12 pts) **NOTE: The two questions below are unrelated.**

(a) Find the tangent line to $y = \dfrac{4\sqrt{x}}{2 - x^3}$ at $x = 1$ and give the $(x, y)$ coordinates at which this tangent line intersects the $x$-axis (as shown below).




(b) Find the $x$ and $y$ coordinates of all points $P$ on the curve $y = \dfrac{1}{x^2}$ at which the tangent lines at the points $P$ have a $y$-intercept of 27.