Math 124 - Fall 2011
Exam 1
October 25, 2011

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

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- There are 5 questions spanning 5 pages. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 80 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 15 MINUTES PER PAGE!

GOOD LUCK!

1. (16 pts) Determine the values of the following limits or state that the limit does not exist. If it is correct to say that the limit equals ∞ or $-\infty$, then you should do so. **In all cases, show your work/reasoning**.

(a) $\displaystyle\lim_{x\to 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x-4}$

(b) $\displaystyle\lim_{x\to 1^-} \frac{|x^2 - x|}{x^2 - 1}$

(c) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{4x^3} + 5x^2}{\sqrt{2x^4 + 7x + 1}}$

(d) $\displaystyle\lim_{x\to 0} \frac{\sin(x) - \tan^{-1}\left(\frac{1}{x^2}\right) + 7}{x}$

2. (a) (7 pts) Let $f(x) = 12\sqrt[3]{x} + \dfrac{2x}{x^6} - x^5 e^{3x}$.
Find the equation for the tangent line to $f(x)$ at $x = 1$.

(b) (7 pts) Let $g(x) = \dfrac{1}{x+1}$. Find a number $x = a$, greater than 1, such that the slope of the **secant** line to $g(x)$ from $x = 1$ to $x = a$ is exactly $-\frac{1}{24}$.

3. (10 pts) To get back at your instructor for a challenging exam you make him take a ride on a giant slingshot at a nearby carnival. Your instructor is attached to two giant bungee cords that launch him vertical straight up, then he oscillates up and down.
Suppose your instructor's height off the ground after, t, seconds is given by

$$y(t) = \frac{50e^{2t} - 40\cos(2t)}{e^{2t}} \quad \text{feet.}$$

(a) (3 pts) Find $\lim_{t\to\infty} y(t)$. (Justify your answer).

(b) (4 pts) Find your instructor's initial velocity.

(c) (3 pts) Find the first positive time when your instructor's velocity is zero.

4. (10 pts) Let a and b be constants. Consider the function $f(x) = \begin{cases} x|x+2| & \text{, if } x < -1; \\ x^{1/3} & \text{, if } -1 \leq x \leq 1; \\ ax + b & \text{, if } x > 1. \end{cases}$

(a) (5 pts) Find a and b so that $f(x)$ is continuous and differentiable at $x = 1$.

(b) (5 pts) Using your values of a and b from part (a), $f(x)$ is now continuous everywhere. However, it is NOT differentiable everywhere. Find all values of x at which $f(x)$ is not differentiable. Justify why the function is not differentiable at the values you give.

5. (10 pts) Consider the parabola $y = x^2$ in the figure below. The triangle is an **equilateral** triangle and the segments $\overline{PQ}$ and $\overline{RP}$ are **normal** (*i.e.* perpendicular) to the parabola at P and R, respectively. Find the coordinates of Q.