Math 124 - Fall 2010
Exam 2
November 23, 2010

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

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- There are 6 questions spanning 5 pages. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 80 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 15 MINUTES PER PAGE!

GOOD LUCK!

1. (12 pts) In each part, find $\frac{dy}{dx}$. Simplify your answers.

(a) $y = \ln(1 + x^4) - \tan^{-1}(x^2)$

(b) $y^3 = (6x)^{(x^2)}$ (put your answer in terms of x)

(c) $x(t) = t\cos(t)$, $y(t) = e^t - t$ (your answer will be in terms of t)

2. (7 pts) Use implicit differentiation to find the equation of the tangent line to the curve

$$x^2 - xy^2 + 1 = (x + y^2)^2$$

at the point $(x, y) = (0, 1)$.

3. (7 pts) Find the absolute maximum and absolute maximum values of $g(x) = 14x^2 - x^4$ on the interval $[-2, 4]$. Justify your answers.

4. (12 points) Consider the function $f(x) = 6x^{4/3} - x^2$. Justify your work in each part using appropriate first and/or second derivative tests.

   (a) Find all critical points of $f(x)$. Classify each critical point as a local max, local min, or neither.

   (b) Find all inflection points of $f(x)$.

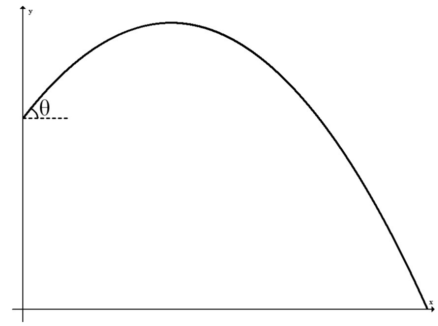
5. (10 pts) A trough is 20 ft long and its ends have the shape of isosceles triangles that are 6 feet across at the top and have a height of 2 feet.

   The trough is placed under a pipe which is leaking out water at a constant rate of c ft^3/min.

   (a) Assume it is known that the water is leaking at a constant $c = 6$ ft^3/min. How fast is the water level rising when the water is 9 inches deep?

   (b) Assume c is not known initially, but it is known to be constant. At time $t = 0$, the trough is empty. Two minutes later, the trough is 6 inches (0.5 feet) deep. Find the constant rate at which the volume of water is leaking, *i.e.* find c.

6. (12 pts)



A pumpkin is fired from a cannon off a cliff and into a corn field. The location of the pumpkin at time t is given by the parametric equations

$$x(t) = 50\cos(\theta)t \quad \text{and} \quad y(t) = 30 + 50\sin(\theta)t - 16t^2,$$

where the angle, θ, is the initial angle at which the pumpkin is fired measured from the horizontal. All distances are in feet and time is in seconds.

(a) If $\theta = \frac{\pi}{4}$ radians, find the time(s) when the horizontal velocity is twice the size of the vertical velocity.

(b) If we wanted to find an angle, θ, in order to make the pumpkin land on a target at (100,0), we would ultimately need to solve the equation (you don't have to derive this):

$$15 + 50\tan(\theta) - 32\sec^2(\theta) = 0.$$

There are two answer between 0 and pi/2 radians and one of the answers is 'near' $\theta = \pi/4$. Find the linear approximation of $f(\theta) = 15 + 50\tan(\theta) - 32\sec^2(\theta)$ at $\theta = \pi/4$. Use the linear approximation to estimate a solution to $f(\theta) = 0$.

(BONUS POINT) One extra credit bonus point if you can give an exact form answer for both angles θ between 0 and $\pi/2$ that solve this equation (put your answer on the back of this page).