**Math 124 Write Up Problem 2: PLAYGROUND MATH**
**DUE WEDNESDAY, NOVEMBER 14th**

Give me organized and presentable solutions that show your work for the following playground related math questions. The goal in this project is threefold: 1. To apply what we have learned, 2. To see parametric equations, related rates and linear approximation in practice, and 3. To have fun! We can argue about whether we have accomplished the last one when you are done. I have tried to accurately model these scenarios, but in some situations I chose mathematical simplicity over modeling accuracy (so don't take all these equations as physical law). Note that problems 2, 3, and 4 are all (very different) examples of circular motion.

Start this project right away. These are like four big homework questions, so you want to spread out your work a bit.

1. **THE SLIDE:** A (frictionless) slide is in the shape of the straight line, $y = mx + b$, with $m < 0$. The top of the slide is at the point, $(0, b)$, with $b > 0$, and the base of the slide is on the $x$-axis. A person with initial speed, $v_0$, in the direction down the slide will have location given by:
$$\begin{aligned} x(t) &= \left(\frac{4.9}{\sqrt{1+m^2}}\right) t^2 + \left(\frac{v_0}{\sqrt{1+m^2}}\right) t \\ y(t) &= \left(\frac{4.9m}{\sqrt{1+m^2}}\right) t^2 + \left(\frac{v_0 m}{\sqrt{1+m^2}}\right) t + b. \end{aligned},$$
where $t$ is in seconds and $x$ and $y$ are in meters.
   (a) By taking appropriate derivatives, verify that the speed of this parameterized motion at $t = 0$ is in fact $v_0$.
   (b) Eliminate the parameter to verify this is indeed motion along the given line. Then, using related rates, find the relationship between $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in general.
   (c) There are two (frictionless) straight line slides in your playground. You and your friend plan to start down your slides at the same time. Here is the information:
   *Friend's Slide*: $b = 5$ m, $v_0 = 0$ m/s, and the slide makes a 30 degree angle with the ground.
   *Your Slide*: $b = 10$ m, $v_0 =$ ??? m/s, and the slide makes a 25 degree angle with the ground.
   Find $v_0$, $x'(0)$ and $y'(0)$ for **you**, so that you and your friend reach the bottom of the side at exactly the same time. (Verify that the correct relationship from part (b) holds between $x'(0)$ and $y'(0)$).

2. **THE MERRY-GO-ROUND:** As your friends keep the merry-go-round spinning at a constant angular speed, you get on at some point and walk at a constant rate directly across the merry-go-round (*i.e.* across the diameter on a straight line). If the origin is the center of the merry-go-round, the parametric equations for your motion are given by:
$$\begin{aligned} x(t) &= r(t)\cos(\theta_0 + \omega t) \\ y(t) &= r(t)\sin(\theta_0 + \omega t) \end{aligned},$$
where $t$ is in seconds, $x$ and $y$ are in meters, $\theta_0$ is the initial angle (in standard position), $\omega$ is angular speed, and $r(t)$ is the signed radius relative to the starting point.
The radius of the merry-go-round is 3 meters and you walk at a constant speed of 1/2 meter per second. Thus, $r(t)$ is a linear function that satisfies $r(0) = 3$, $r(6) = 0$, and $r(12) = -3$.
   (a) Find the formula for $r(t)$.
   (b) Consider the scenario where $\theta_0 = 0$ and the merry-go-round rotates once every 14 seconds.
      i. Draw the graph of the motion for $0 \le t \le 12$. Label all $x$ and $y$ intercepts and indicate the time at these points. Also, label the exit point.
      ii. Find the equation of the tangent line to the path of motion at the point (x,y)=(0,0).
   (c) If the merry-go-round rotates once every 5 seconds, at what point $(x, y)$ should you get on the merry-go-round in order to end up at the point $(0, -3)$ when you get off?

3. **THE SWINGS:** Let $h = 15$ feet be the height of the top of the swing-set and $r = 11$ feet be the length of the swing. If we impose a coordinate system with the ground under the swing set as the origin, then we will assume the parametric equations for motion are given by:

$$\begin{aligned} x(t) &= 11\cos(\tfrac{3\pi}{2} + \theta(t)) \\ y(t) &= 15 + 11\sin(\tfrac{3\pi}{2} + \theta(t)) \end{aligned},$$

where $t$ time in seconds and $\theta(t)$ is a function for the angle counterclockwise from the vertical at time $t$.

The swing starts with an initial angle of $\theta(0) = -\frac{\pi}{6}$ radians (-30 degrees) and reaches it's low point for the first time at $t = 1/2$ seconds.

(a) Use related rates and the fact that $x$ and $y$ are points on a circle to find a relationship between $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

(b) (Model 1) Assume $\theta(t) = a\cos(bt)$ for some constants $a$ and $b$.

i. Find $a$ and $b$.

ii. Find $x(2)$, $y(2)$, $x'(2)$ and $y'(2)$. You should use part (a) to check your work. Describe in words what is happening at $t = 2$. Then describe the long term behavior of this model (*i.e.* what is happening as $t \to \infty$).

(c) (Model 2) Assume $\theta(t) = a\cos(bt)e^{-ct}$ for some constants $a$, $b$, and $c$.

i. In addition to the assumptions above, also assume $\theta(1) = \frac{\pi}{8}$. Find $a$, $b$, and $c$.

ii. Describe the long term behavior of this model (*i.e.* what is happening as $t \to \infty$).

4. **THE SEE-SAW:** Your pet elephant, Elle, and your pet mouse, Mickey, plan to play on the see-saw. The fulcrum is 0.5 meter off the ground and directly in the middle of the see-saw which is of length $2r$ ($r$ from fulcrum to end). The mouse gets on the low side of the see-saw and the elephant steps from a platform onto the high side (so the initial velocity of the Elephant is zero). If we impose a coordinate system with the origin on the ground under the fulcrum, then the equations for the elephant's motion are:

$$\begin{aligned} x(t) &= r\cos(\theta(t)) \\ y(t) &= 0.5 + r\sin(\theta(t)) \end{aligned},$$

where $r$ is in meters, $t$ is in seconds, $\theta(t)$ is the angle of the see-saw measure from horizontal.

If $\theta_0 = \pi/10$ radians (18 degrees) is the initial angle of the see-saw and if we assume that $\theta(t)$ has constant acceleration $b$, then we have $\theta(t) = \frac{\pi}{10} + \frac{b}{2}t^2$. Give answers accurate to 5 digits after the decimal.

(a) Find $r$.

(b) The arc length swept out by the motion of the elephant is given by $s = r\theta$. Assume $\frac{d^2s}{dt^2} = -9.8$ m/s$^2$ (which is only approximately correct). Use related rates to give the value of $b = \frac{d^2\theta}{dt^2}$.

(c) Find the time when the elephant reaches the ground.

(d) The instant the elephant's side hits the ground, the mouse is launched in the air on a parabolic path.
Let $(A, B) =$ the point at which the mouse starts it's launched into the air.
Let $C =$ the horizontal velocity of the mouse at the instant the elephant hits the ground.
Let $D =$ the vertical velocity of the mouse at the instant the elephant hits the ground.
The equations for the motion of the mouse after it is launched in the air are:

$$\begin{aligned} x(t) &= A + Ct \\ y(t) &= B + Dt - 4.9t^2 \end{aligned}.$$

Find $A$, $B$, $C$, and $D$.

(e) What is the maximum height the mouse reaches?