IN ALL CASES FIND $\frac{dy}{dx}$

Derivatives Practice

With the techniques we have developed, we can now differentiate almost any functions we encounter by using some combination of known rules. Below is a large collection of derivatives each pulled directly from the old exams archives.

1. $y = (\ln(x))^3$

CHAIN RULE

$y' = 3(\ln(x))^2 \cdot \frac{1}{x}$

$\boxed{y' = \frac{3(\ln(x))^2}{x}}$

2. $y = x^{\cos(x)}$

LOG. DIFF.

$\frac{d}{dx}\left[\ln(y) = \cos(x)\ln(x)\right]$

$\frac{1}{y}y' = \cos(x)\cdot\frac{1}{x} - \sin(x)\ln(x)$

$y' = y\left(\frac{\cos(x)}{x} - \sin(x)\ln(x)\right)$

$\boxed{y' = x^{\cos(x)}\left(\frac{\cos(x)}{x} - \sin(x)\ln(x)\right)}$

3. $y = \tan(e^x)$

CHAIN RULE

$y' = \sec^2(e^x)\cdot e^x$

$\boxed{y' = e^x \sec^2(e^x)}$

4. $\frac{d^{16}}{dx^{16}}(3x^{15})$

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Ex) $\frac{d^3}{dx^3}(x^2)$

$= \frac{d^2}{dx^2}(2x)$

$= \frac{d}{dx}(2) = 0$

5. $x^4 + xy + y^4 = 3$

IMPLICIT DIFF

$4x^3 + y + xy' + 4y^3y' = 0$

$(x+4y^3)y' = -4x^3 - y$

$y' = \frac{-4x^3-y}{x+4y^3}$

$\boxed{y' = -\frac{(y+4x^3)}{(x+4y^3)}}$

6. $y = 7(x^2 + 3x)$

EXPAND FIRST

$y = 7x^2 + 21x$

$\boxed{y' = 14x + 21}$

7. $y = \cos(\tan(2x))$

CHAIN RULE

$y' = -\sin(\tan(2x))\sec^2(2x)\cdot 2$

$\boxed{y' = -2\sin(\tan(2x))\sec^2(2x)}$

8. $y = (ax^2 + b)e^{-cx}$

PRODUCT RULE

$y' = (ax^2+b)(-ce^{-cx}) + 2axe^{-cx}$

$\boxed{y' = (-acx^2 - bc + 2ax)e^{-cx}}$

9. $y = \frac{3x\ln(x)}{2x^3 - x + 7}$

QUOTIENT RULE

$y' = \frac{(2x^3-x+7)(3x\cdot\frac{1}{x} + 3\ln(x)) - (6x^2-1)(3x\ln(x))}{(2x^3-x+7)^2}$

$\boxed{y' = \frac{(2x^3-x+7)(3+3\ln(x)) - (6x^2-1)(3x\ln(x))}{(2x^3-x+7)^2}}$

10. $y = (\sin(x))^{x^2}$

LOG. DIFF.

$\frac{d}{dx}\left[\ln(y) = x^2\ln(\sin(x))\right]$

$\frac{1}{y}y' = 2x\ln(\sin(x)) + x^2\frac{\cos(x)}{\sin(x)}$

$y' = y\left(2x\ln(\sin(x)) + x^2\cot(x)\right)$

$\boxed{y' = (\sin(x))^{x^2}\left(2x\ln(\sin(x)) + x^2\cot(x)\right)}$

11. $y^3 - x^2y - 2x^3 = 8$

IMPLICIT DIFF.

$3y^2y' - x^2y' - 2xy - 6x^2 = 0$

$(3y^2 - x^2)y' = 2xy + 6x^2$

$\boxed{y' = \frac{2xy+6x^2}{3y^2-x^2}}$

12. $y = 240\ln(x/12) - 20x$

SIMPLIFY FIRST (or USE CHAIN RULE)

$y = 240\ln(x) - 240\ln(12) - 20x$

$\boxed{y' = \frac{240}{x} - 20}$

13. $y = \left(\frac{x^2+1}{x^4+2}\right)^{50}$

CHAIN RULE (OR LOG. DIFF.)

$\ln(y) = 50\ln\left(\frac{x^2+1}{x^4+2}\right)$

$\frac{d}{dx}\left[\ln(y) = 50\ln(x^2+1) - 50\ln(x^4+2)\right]$

$\frac{1}{y}y' = \frac{50\cdot 2x}{x^2+1} - \frac{50\cdot 4x^3}{x^4+2}$

$y' = y\left(\frac{100x}{x^2+1} - \frac{200x^3}{x^4+2}\right)$

$\boxed{y' = \left(\frac{x^2+1}{x^4+2}\right)^{50}\left(\frac{100x}{x^2+1} - \frac{200x^3}{x^4+2}\right)}$

14. $y = \left(\frac{2}{x}\right)^{1/x}$

LOG DIFF

$\ln(y) = \frac{1}{x}\ln(2/x)$

$\frac{d}{dx}\left[\ln(y) = \frac{1}{x}\ln(2) - \frac{1}{x}\ln(x)\right]$

$\frac{1}{y}y' = -\frac{\ln(2)}{x^2} - \frac{1}{x}\cdot\frac{1}{x} + \frac{1}{x^2}\ln(x)$

$y' = y\left(-\frac{\ln(2)}{x^2} - \frac{1}{x^2} + \frac{1}{x^2}\ln(x)\right)$

$\boxed{y' = \left(\frac{2}{x}\right)^{1/x}\left(-\ln(2) - 1 + \ln(x)\right)\frac{1}{x^2}}$

15. $y = \cos^2(\sin(x))$

CHAIN RULE

$y' = 2\cos(\sin(x))(-\sin(\sin(x)))\cos(x)$

$\boxed{y' = -2\cos(\sin(x))\sin(\sin(x))\cos(x)}$

16. $x^3 - y^3 = 2xy$

IMPLICIT DIFF

$3x^2 - 3y^2y' = 2xy' + 2y$

$3x^2 - 2y = 2xy' + 3y^2y'$

$3x^2 - 2y = (2x + 3y^2)y'$

$\boxed{y' = \frac{3x^2-2y}{3y^2+2x}}$

17. $y = \frac{3x^2 + \ln(x)}{\sin(e^x)}$

QUOTIENT RULE

$\boxed{y' = \frac{\sin(e^x)(6x+\frac{1}{x}) - (3x^2+\ln(x))\cos(e^x)e^x}{\sin^2(e^x)}}$

18. $y = x\tan^{-1}(\sqrt{x})$

PRODUCT RULE / CHAIN RULE

$y' = \tan^{-1}(\sqrt{x}) + \frac{x}{(1+(\sqrt{x})^2)}\,\frac{1}{2\sqrt{x}}$

$\boxed{y' = \tan^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{2(1+x)}}$

22. $y=\cos(e^{-x^2})$

CHAIN RULE

$y' = -\sin(e^{-x^2})e^{-x^2}(-2x)$

$$\boxed{y' = 2xe^{-x^2}\sin(e^{-x^2})}$$

23. $y = x^{\sin(2ax)}$

LOG. DIFF.

$\ln(y) = \sin(2ax)\ln(x)$

$\frac{1}{y}y' = \sin(2ax)\frac{1}{x} + 2a\cos(2ax)\ln(x)$

$$\boxed{y' = x^{\sin(2ax)}\left(\frac{\sin(2ax)}{x} + 2a\cos(2ax)\ln(x)\right)}$$

24. $y = \frac{x}{\sqrt{1-x^2}}$

QUOTIENT RULE

$y' = \dfrac{\sqrt{1-x^2}\cdot 1 - x\,\frac{-2x}{2\sqrt{1-x^2}}}{(\sqrt{1-x^2})^2}$

$y' = \dfrac{\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}}}{1-x^2}$

$y' = \dfrac{1-x^2+x^2}{(1-x^2)\sqrt{1-x^2}}$

$$\boxed{y' = \frac{1}{(1-x^2)^{3/2}}}$$

25. $\sin(x+2y) = 2x\cos(y)$

IMPLICIT DIFF.

$\cos(x+2y)(1+2y') = 2\cos(y) - 2x\sin(y)y'$

$\cos(x+2y) + 2\cos(x+2y)y' = 2\cos(y) - 2x\sin(y)y'$

$(2\cos(x+2y) + 2x\sin(y))y' = 2\cos(y) - \cos(x+2y)$

$$\boxed{y' = \frac{2\cos(y) - \cos(x+2y)}{2x\sin(y) + 2\cos(x+2y)}}$$

26. $y = \dfrac{\cos(x^2)e^{x^2-x}}{\ln(2x-3)}$

LOG DIFF (OR QUOTIENT)

$\ln(y) = \ln(\cos(x^2)) + \ln(e^{x^2-x}) - \ln(\ln(2x-3))$

$\frac{d}{dx}[\ln(y) = \ln(\cos(x^2)) + x^2 - x - \ln(\ln(2x-3))]$

$\frac{1}{y}y' = \frac{-2x\sin(x^2)}{\cos(x^2)} + 2x - 1 - \frac{2}{(2x-3)\ln(2x-3)}$

$$\boxed{y' = \frac{\cos(x^2)e^{x^2-x}}{\ln(2x-3)}\left(-2x\tan(x^2) + 2x - 1 - \frac{2}{(2x-3)\ln(2x-3)}\right)}$$

27. $y = \tan^{-1}(\ln(x^3+5))$

CHAIN RULE

$$\boxed{y' = \frac{3x^2}{(1+\ln(x^3+5)^2)(x^3+5)}}$$

28. $y = \sin\left(\dfrac{e^t}{\cos(t)}\right)$

CHAIN RULE (AND QUOTIENT)

$y' = \cos\left(\frac{e^t}{\cos(t)}\right)\left(\frac{\cos(t)e^t - e^t(-\sin(t))}{\cos^2(t)}\right)$

$$\boxed{y' = e^t\cos\left(\frac{e^t}{\cos(t)}\right)\underbrace{\left(\frac{\cos(t)+\sin(t)}{\cos^2(t)}\right)}_{\text{sam as } \sec(x)+\sec(x)\tan(x)}}$$

29. $y = x^{x\sec(x)}$

LOG. DIFF.

$\frac{d}{dx}[\ln(y) = \underbrace{x\sec(x)}_{F}\underbrace{\ln(x)}_{S}]$

$\frac{1}{y}y' = x\sec(x)\frac{1}{x} + \ln(x)(\sec(x) + x\sec(x)\tan(x))$

$$\boxed{y' = x^{x\sec(x)}(1 + \ln(x) + x\ln(x)\tan(x))\sec(x)}$$

30. $\frac{x}{y} + x^y = 5$

IMPLICIT AND LOG

$x^y = 5 - x/y$ isolate exponential

$yx^y = 5y - x$ simplify

$\ln(yx^y) = \ln(5y-x)$

$\frac{d}{dx}[\ln(y) + y\ln(x) = \ln(5y-x)]$

$\frac{1}{y}y' + y'\ln(x) + \frac{y}{x} = \frac{5y'-1}{5y-x}$

$y'(5y-x)\left(\frac{1}{y} + \ln(x)\right) + \frac{y}{x}(5y-x) = 5y' - 1$

$y'[(5y-x)(\frac{1}{y} + \ln(x)) - 5] = -1 - \frac{y}{x}(5y-x)$

$y'[5 + 5y\ln(x) - \frac{x}{y} - x\ln(x) - \cancel{5}] = -1 - \frac{y}{x}(5y-x)$

$$\boxed{y' = -\frac{(1 + 5y^2/x - y)}{(5y\ln(x) - \frac{x}{y} - x\ln(x))}}$$