Math 124 - Fall 2011
Exam 2
November 22, 2011

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

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- There are 6 questions (with parts) spanning 5 pages. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 80 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 15 MINUTES PER PAGE!

GOOD LUCK!

1. (12 pts) Compute the slopes of the tangents at the specified points. Give your final answers simplified in exact form.

   (a) Let $f(x) = \ln(\tan^{-1}(x^3))$. Find the slope of the tangent line at $x = 1$.

   (b) Let $y = \left(e^{2x} - \ln(x)\right)^{10}$. Find the slope of the tangent at $x = 1$.

   (c) Let $y = \frac{1}{2}(x)^{\sqrt{x}}$. Find the slope of the tangent line at $x = 4$.

2. (7 pts) Consider the implicitly defined curve given by: $x^2 + y^2 = \cos(x) + 3xy^3$.

   This curve has two y-intercepts. Find the **equations of the tangent lines** at each y-intercept.

3. (7 pts) Find the absolute maximum and minimum values of $f(x) = x^2 - 3\ln(x^2+1)$ on the interval $[-1, 3]$. Justify your answer.

4. (10 pts) Suppose we want to approximate the solution(s) to $\cos(x) = x$.
We let $f(x) = \cos(x) - x$.

   (a) Precisely explain in words why $f(x) = 0$ has exactly one solution between $x = 0$ and $x = \pi$. Your explanation should contain 2-3 sentences. As part of your explanation, find $f'(x)$.

   (b) Through experimentation, you find the solution is near $x = \frac{\pi}{4}$. Find the linear approximation to $f(x) = \cos(x) - x$ at $x = \frac{\pi}{4}$ and use it to approximate the solution to $f(x) = 0$. In other words, do the first step of Newton's method. (Give your final answer as a decimal to five digits.)

5. (12 pts) Sand is being dumped from a conveyor belt at a rate of 12π ft^3/min, forming a conical pile of sand. Four minutes later, the radius of the base of the pile is 6 feet and the height is 4 feet, and the radius is increasing at a rate of $\frac{1}{4}$ ft/min.
(Recall: The volume of a cone is $V = \frac{\pi}{3}r^2h$).

   (a) How fast is the height changing four minutes after the pile starts?

   (b) Viewed from the side, the conical pile is an isosceles triangle with base of length $2r$ and the other two sides of equal length. How fast are the other side lengths of the triangle changing four minutes after the pile starts?

6. (12 pts) You are on a ferris wheel. Let the origin be the location where the ferris wheel is attached to the ground. You start keeping track of your location (at $t = 0$ seconds) and you find that as the wheel rotates, your coordinates (in feet) are given by the parametric equations

$$x(t) = 70\cos\left(\frac{\pi}{15}t\right)$$
$$y(t) = 70\sin\left(\frac{\pi}{15}t\right) + 74$$

(a) (3 pts) Give the following information:

Radius of the Wheel $=$ ____________________

Initial, $t = 0$, location (x, y) $=$ ____________________

Time it takes to complete one revolution $=$ ____________________

(b) (5 pts) Find the slope of the tangent at $t = 10$ seconds.

(c) (4 pts) At the instant when you reach the far left point on the ferris wheel, you 'accidentally' throw a water balloon directly downward. At $t = 1$ seconds after the instant you threw the balloon, it lands on Dr. Loveless' head who happens to be standing on the ground below. Dr. Loveless is 6 feet tall.

The balloon's height, t, seconds after being thrown is given by $h(t) = 74 - At - 16t^2$, where A is the magnitude of the initial vertical velocity of the balloon (the vertical speed of the ferris wheel plus the speed at which your arm threw the balloon).

At what initial speed did your arm throw the balloon?