My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

3.3: Derivatives of Trig Functions

1. Know the derivatives (put this somewhere and look at it often, you want to be able to call upon these from memory):

$\frac{d}{dx}[\sin(x)] = \cos(x)$	$\frac{d}{dx}[\cos(x)] = -\sin(x)$
$\frac{d}{dx}[\tan(x)] = \sec^2(x)$	$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$
$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$	$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$

2. Understand graphically how to recall these derivatives as well. You should also know how to derive the derivatives for $\tan(x)$ and $\sec(x)$ from $\sin(x)$ and $\cos(x)$ (and similarly for $\cot(x)$ and $\csc(x)$).

3. In the derivation the following limits were useful which you are welcome to quote as fact in the future:

- $\lim\limits_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$

- $\lim\limits_{\theta \to 0} \frac{\cos(\theta) - 1}{\theta} = 0$

4. When working with trigonometric functions it is useful to have a handle on some of the basic identities. So please permit me to give a brief discussion of a way to remember a large number of the identities. Many of the identities can be derived from the sum identities:

(a) $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$

(b) $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

(c) Here are some special cases:

- With $\alpha = \beta = \theta$ in (b) with a minus, we get
$$1 = \sin^2(\theta) + \cos^2(\theta).$$

- Dividing the above by $\cos^2(\theta)$ gives
$$\sec^2(\theta) = \tan^2(\theta) + 1.$$

- With $\alpha = \beta = \theta$ in (a) with a plus, we get
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta).$$

- With $\alpha = \beta = \theta$ in (b) with a plus, we get
$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta).$$

5. More trig facts (including various values, circular motion, sinusoidal wave information, and more can be found on the pre-calculus review sheet that you received the first day of class which is also our course website). There is also a discussion of trigonometric functions in Appendix D of your book which you may find helpful. If you can find your answers in my review or the book review, or if you just need a refresher, then please feel free to come ask me at office hours.

3.4: The Chain Rule

1. The **Chain Rule** is the rule for differentiating a function of the form $f(g(x)) = (f \circ g)(x)$ (a function 'inside' another function). It is a rule of vital importance which we will use almost daily as we differentiate. The chain rule says:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

If we label $y = f(u)$ and $u = g(x)$, we can write this relationship in Leibniz notation as:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

2. If you have difficulty applying the chain rule to $f(g(x))$, it may be useful to actually label the 'inside' function by $u = g(x)$, then find $f'(u)$ and $g'(x)$. Multiply the result and replace u by $g(x)$.

3. Special Cases (it is much, much better to understand the general rule as opposed to memorizing a bunch of special cases, I simply give these here so you can see the chain rule in action):

   (a) The General Power Rule:

   $$\frac{d}{dx}[(g(x))^n] = n(g(x))^{n-1}g'(x).$$

   This includes the situation

   $$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \frac{d}{dx}\left[(g(x))^{-1}\right] = -(g(x))^{-2}g'(x) = -\frac{g'(x)}{[g(x)]^2},$$

   which gives you another approach to some quotient rule problems.

   (b) The General Exponential Rule:

   $$\frac{d}{dx}\left[e^{g(x)}\right] = e^{g(x)}g'(x).$$

   (c) Some General Trig Rules:

   $$\frac{d}{dx}[\sin(g(x))] = \cos(g(x))g'(x)$$
   $$\frac{d}{dx}[\cos(g(x))] = -\sin(g(x))g'(x)$$
   $$\frac{d}{dx}[\tan(g(x))] = \sec^2(g(x))g'(x)$$
   $$\frac{d}{dx}[\sec(g(x))] = \sec(g(x))\tan(g(x))g'(x).$$

   $$etc....$$

4. It is not unusual (in fact quite common) that we will have to use the chain rule several times in a derivative. For example: $f(x) = \cos(\sin(e^{3x}))$ will require several uses of the chain rule. We simply start from the 'outside' function and work out way in as follows:

$$\frac{d}{dx}\left[\cos(\sin(e^{3x}))\right] = -\sin(\sin(e^{3x})) \cdot \cos(e^{3x}) \cdot e^{3x} \cdot 3$$

5. You can see why we use the name 'chain rule' as we are multiplying together and building a chain of derivatives.