

Week 3 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

2.7 and 2.8: The Derivative

- Understand how we use slopes of secant lines to approximate the slope of the tangent and we defined:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \text{the slope of the tangent line to } f(x) \text{ at } a.$$

- If the limit exists at a , we say the function is differentiable at $x = a$. We discussed how a function is not differential at (i) discontinuities, (ii) at 'sharp corners', and (iii) when the tangent is vertical.
- We defined various notations for differentiation and we discussed higher derivatives.
- The units of $f'(a)$ are the units of the rate of $\frac{\text{units of } y=f(x)}{\text{units of } x}$.
- If we treat x as a variable, then $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ defines a new function.
- The graphs of $y = f(x)$ and $y = f'(x)$ can look very different (note in particular that these y 's are totally different quantities with different units), but there are strong connections between them. These are important connections to understand visually as we will use them throughout the quarter.
 - $f'(x) = 0$ (that is, the graph of the derivative crosses the x -axis) precisely when $f(x)$ has a horizontal tangent (a hill or valley).
 - $f'(x) > 0$ (that is, the graph of the derivative is above the x -axis) precisely when $f(x)$ is increasing (going uphill from left to right).
 - $f'(x) < 0$ (that is, the graph of the derivative is below the x -axis) precisely when $f(x)$ is decreasing (going downhill from left to right).
 - $f'(x)$ is largest positive precisely when $f(x)$ is increasing the fastest (steepest increase).
 - $f'(x)$ is largest negative precisely when $f(x)$ is decreasing the fastest (steepest decrease).

3.1 and 3.2: Derivative Rules and Shortcuts

- Here is a quite list of the derivative rules for basic functions we pick up in 3.1. Note that c is a constant.

$\frac{d}{dx} [c] = 0$ (Constant Rule)	$\frac{d}{dx} [x^n] = nx^{n-1}$ (Power Rule)	$\frac{d}{dx} [e^x] = e^x$ (Exponential Rule)
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- Here are some important derivative function arithmetic rules we picked up in 3.1 and 3.2. Again c is a constant.

$\frac{d}{dx} [cf(x)] = c \frac{d}{dx} [f(x)]$	(Pull out constant)
$\frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} [f(x)] \pm \frac{d}{dx} [g(x)]$	(Sum/Difference Rule)
$\frac{d}{dx} [f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$	(Product Rule)
$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$	(Quotient Rule)

3. Understand how to calculate derivatives by:

- (a) Rewriting/Simplifying the given function in a form so that the rules apply (need to be able to handle: roots, fractional powers, negative powers, products, fractions, *etc.*)
- (b) Use the rules.
- (c) Simplify your final answer is a nice form.

4. Visually using the derivative:

- (a) Recall: $f'(a)$ = the slope of the tangent line at to $f(x)$ at the point $(a, f(a))$. Using the known equation for a **tangent line** we get:

$$\text{The equation for the tangent line to } f(x) \text{ at } x = a : y = f'(a)(x - a) + f(a).$$

- (b) The **normal line** to $f(x)$ at $x = a$ is the line through $(a, f(a))$ that is perpendicular to the tangent (so it has slope $-1/f'(a)$). Thus, the equation of the normal line is given by:

$$\text{The equation for the normal line to } f(x) \text{ at } x = a: y = -\frac{1}{f'(a)}(x - a) + f(a).$$

- (c) The simpler questions in the homework give you the point $(a, f(a))$ and ask you to find the tangent slope, tangent line, normal slope or normal line (for all of these you start by finding $f'(a)$ and then you plug in what you know).
- (d) The more interesting questions in the homework give you information about the tangent or normal and ask you to find the point $(a, f(a))$. For these problems you should:
 - Give your unknown point a name: that is label it $(a, f(a))$. (This is always a good problem solving strategy)
 - Symbolically find $f'(a)$.
 - If you are given the desired tangent slope, solve $f'(a) =$ 'the desired tangent slope'. If you are given the desired normal slope, solve $f'(a) =$ 'the desired normal slope'. That is, symbolically write down what is you are being asked to find and solve for a .