Exam 2
February 23, 2016
Name: $\qquad$

Section: $\qquad$
Student ID Number: $\qquad$

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- There are 6 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (no other calculators allowed). And you are allowed one hand-written 8.5 by 11 inch page of notes (front and back).
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.
- Give exact answers wherever possible. Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. For example, don't leave your answer in the form $\sqrt{4}$ or $\cos (\pi / 4)$ instead write $\sqrt{4}=2$ and $\cos (\pi / 4)=\sqrt{2} / 2$. But otherwise, you do not have to simplify.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the test. Cheating will not be tolerated. We report all suspicions of cheating to the academic misconduct board. If you are found guilty of cheating by the misconduct board, then you will get a zero on the exam (and likely face other academic penalties). Keep your eyes on your exam!
- You have 80 minutes to complete the exam. Budget your time wisely.

SPEND NO MORE THAN 10 MINUTES PER PAGE!

1. (a) (4 pts) Given $f(x)=\left(\sin ^{2}(x)+e^{\left(5 x^{4}\right)}\right)^{10}$, find $f^{\prime}(x)$.
(b) (5 pts) Given $y=\left(x^{2}+1\right)^{(\cos (4 x))}$, find $\frac{d y}{d x}$.
(c) (6 pts) Find the equation of the tangent line to $y^{3}+y \sin (x)=\cos (x)$ at the point on the curve where $x=0$.
2. (7 pts) Use the linear approximation to $f(x)=\tan ^{-1}(2 x)+\ln \left(8 x^{3}\right)$ at $x=\frac{1}{2}$ to estimate the value of $f(0.51)$. (Leave in exact form).
3. $(7 \mathrm{pts})$ Consider the curve implicitly defined by $\left(x^{3}-y^{2}\right)^{2}+e^{y}=4$ (shown below).

Find the $(x, y)$ coordinates of the point $A$ shown which is the highest point on the curve.
Hint: At this point, there is a horizontal tangent line. (Leave in exact form).

4. (11 pts) For BOTH parts below, consider the parametric curve shown which is defined by:

$$
x(t)=t^{3}-4 t \quad, \quad y(t)=2 \ln \left(t^{2}+1\right)-1
$$

(a) The tangent line is vertical at the point $B$ shown in the graph. Find the $y$-coordinate of the point $B$. (Leave in exact form)

(b) The curve has one positive $y$-intercept which it crosses through twice. Find the equation of the tangent line that has negative slope at the positive $y$-intercept (shown in the picture).
5. ( 8 pts ) An inverted cone starts full of water. The height of the cone is 6 ft and the radius is 4 ft . Water leaks out of the bottom at a constant rate of $1 \mathrm{ft}^{3} / \mathrm{min}$. When the radius is 2 ft , find the rate at which the radius is changing. Recall: The volume of a cone is $V=\frac{1}{3} \pi r^{2} h$. (Give appropriate units and signs, and leave in exact form).
6. (12 pts)

Two carts, $A$ and $B$, are connected by a rope 15 ft long that passes over a pulley $P$ (see the figure). The point $Q$ is on the floor 4 ft beneath $P$ and between the carts. Cart $A$ is being pulled away from $Q$ at a constant speed of $2 \mathrm{ft} / \mathrm{s}$.

(Give appropriate units and signs, and leave in exact form)
(a) Let $\theta$ be the angle that the rope makes with the ground where it meets cart A (as shown in the picture). Find the rate at which $\theta$ is changing at the instant when cart $A$ is 3 ft from $Q$.
(b) How fast is cart $B$ moving toward $Q$ at the instant when cart $A$ is 3 ft from $Q$ ?

