Math 124 - Fall 2012
Exam 2
November 20, 2012

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

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- There are 5 pages of questions. Make sure your exam contains all these pages.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 80 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 15 MINUTES PER PAGE!

GOOD LUCK!

1. (a) (6 pts) Find the equation for the tangent line to the curve $y = \sqrt{e^{\sin(x)} + \ln(5x + 1) + 1}$ at $x = 0$.

(b) (6 pts) At $x = 0.3$ there is only one corresponding y value on the curve implicitly defined by

$$y^5 - x = yx^2 + 1.$$

Use the tangent line approximation at the point $(0, 1)$ to estimate the value of y that corresponds to $x = 0.3$ on this curve.

2. (a) (5 pts) Let $f(x)$ be a function such that it's derivative satisfies $2 \leq f'(x) \leq 5$ for all real values of x. Assuming $f(0) = 1$ and x is positive, by correctly stating and using the mean value theorem on the interval $[0, x]$ give an upper and lower bound on $f(x)$.
(Note: Your bounds will be in terms of x).

(b) (8 pts) Find and classify all critical numbers for $f(x) = \tan^{-1}(x^2) - \frac{1}{8}\ln(x^4 + 1)$.

3. (a) (6 pts) Consider the function $f(x) = \dfrac{e^x}{ax^2+b}$ where a and b are positive constants. Find general conditions on a and b under which $f(x)$ will have two (real) critical numbers and find these two critical numbers.

(b) (7 pts) Find the absolute max and min values of $f(x) = x^{(-1/x^2)}$ on the interval $[\frac{1}{2}, 2]$.

4. At time $t = 0$ min, you start pumping milk into a cone at a constant rate of 0.4 ft^3/min. The bottom point of the cone is dripping, so that the cone is also losing volume at a constant, but unknown, rate, of c ft^3/min. The cone is 8 feet high with a radius of 2 feet at the top.
Recall: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

(a) (7 pts) At some particular time you measure the height of the milk in the cone to be 2 feet with the height increasing at a rate of $\frac{1}{3}$ ft/min. Find the rate, c, at which the milk is dripping out of the funnel.

(b) (4 pts) You put a cylindrical bucket under the cone to catch the drips. The cylindrical bucket is 2 feet high and has a radius of 1/2 foot. At what time, t, is the bucket full?
(This is when you dumped the bucket of milk on Dr. Loveless' head).

5. Since you know that Dr. Loveless has motion sickness, you and some classmates tie him to the edge of a merry-go-round which happens to be on a moving train. The merry-go-round has a radius of 4 feet and is rotating counterclockwise at a constant rate of 5 revolutions per hour. At time $t = 0$, Dr. Loveless is on the northernmost edge of the merry-go-round.

   The train moves at a constant speed in such a way that the **center** of the merry-go-round is at the origin at time $t = 0$ and at the point $(4000, 6000)$ at time $t = 3$ hours.

   The model for this motion is

$$x(t) = at + 4\cos(\theta_0 + wt)$$
$$y(t) = bt + 4\sin(\theta_0 + wt)$$

   (a) (5 pts) Find the constants θ_0, w, a, and b.

   (b) (6 pts) At $t = 4.5$ hours, Dr. Loveless comes untied and falls off the merry-go-round. Find the equation for the tangent line path he follows at $t = 4.5$ hours.