Math 124 Precalculus Review and Worksheet

Lines: For our purposes and for speed, we will prefer the point-slope form of the line equation. That is, $y = m(x - x_1) + y_1$.

- Find the equation for the line through the points $(1, 4)$ and $(5, 6)$. Write it in point-slope form and in slope-intercept form. Which do you think is faster?

Inverses: In order to solve equations you need a good grasp of functions and their inverses. You must know the inverses and how they work addition/subtration, multiplcation/division, powers/roots, exponentials/logarithms, trig/inverse trig. And you must know order of operations. Here are some that look messy, but are easy to solve if you know your inverses.

- Solve $10(e^{\sqrt{x}+1} - 2) = 50$

- Solve $\dfrac{1}{\sqrt[3]{(\ln(x) - 4)^2 + 7}} = 6$

Special notes:

1. There are two possible solutions when taking even roots. For example, the solution to $x^2 = 9$ could be $x = -3$ or $x = 3$. Similarly the solution to $x^4 = 16$ could be $x = -2$ or $x = 2$.

2. Your always looking to get all the x's to the same side and combine like terms. The only special case is when you are looking to factor or use the quadratic formula.

3. If there are fractions, always start by clearing the denominators. For example to solve $1 = \frac{4}{x^2}$, the first thing I do is multiply both sides by x^2 which gives $x^2 = 4$. Another example, to solve $\frac{1}{3x} - 5x = \frac{1}{2}$, I would multiply by $6x$ which would give $2 - 30x^2 = 3x$, which is equivalent to $0 = 30x^2 + 3x - 2$ (a good time to use the quadratic formula).

4. When solving equations involving powers/roots, exp/log, trig/inverse trig, first isolate the 'difficult' function, then use the inverse. For example to solve $\sqrt{x} + x = 1$, we need to get rid of the square root, but in order to do that I first need to isolate it, which gives $\sqrt{x} = 1 - x$. Now we can square both sides to get $x = (1 - x)^2$, which gives $x = x^2 - 2x + 1$, which simplifies to $0 = x^2 - 3x + 1$.

5. When solving equations involving trig, it is typically smart to draw a picture of the corresponding periodic function and to work from the graph. For example to solve $\sin(x) = 0.4$, we can use $x = \sin^{-1}(0.4) \approx 0.411517$ (make sure your in radians), but this is only one solution (the principal solution) to the equation. There are infinitely many more. How do we get the other ones? Use the graph!

   (a) So the principal solution to $\sin(x) = 0.4$ is 0.411517.

   (b) Since the graph of $\sin(x)$ repeats every 2π, we know there will also be solutions at $\ldots 0.411517 - 4\pi, 0.411517 - 2\pi, 0.411517, 0.411517 + 2\pi, 0.411517 + 4\pi, \ldots$. Stated more simplify, one collection of solutions would be $x = 0.411517 + 2k\pi$ for any integer k (an integer is a positive, negative, or zero whole number).

   (c) But that is not all the solutions! There are also what we call 'symmetry' solutions. Look at the graph and you'll see them. Since $\sin(x)$ is symmetric about the $x = \frac{\pi}{2}$ (the peak of the first wave), another solution would be on the other side of this peak. So $\frac{\pi}{2} - 0.411517 \approx 1.159279$ is the distance from the prinipal solution to the first peak, so by symmetry the solution on the other side of the peak must be at $\frac{\pi}{2} + 1.159279 \approx 2.730076$. This solution will also repeat every 2π along the function.

(d) So the general set of solutions to $\sin(x) = 0.4$ would be $x = 0.411517 + 2k\pi$ or $x = 2.730076 + 2k\pi$ for any integer k.

This takes a few moments to do, but I think it is always wise to sketch a quick picture and use symmetry rather than try to memorize a procedure of when to add π or 2π. For $\cos(x)$ there is symmetry at $x = 0$ so the set up is even simpler.
So let's say we want to solve

$$2\cos(x^3) = 1.$$

Isolate the trig function gives $\cos(x^3) = \frac{1}{2}$.
Now we play the trig game from above with the cosine wave.
The principal solution is $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ (we should know this). By symmetry of the cosine wave we also know that $-\frac{\pi}{3}$ is a solution. And we know these solutions repeat every 2π, so we conclude:

$$\cos(x^3) = \frac{1}{2} \text{ implies } x^3 = \frac{\pi}{3} + 2k\pi \text{ or } x^3 = -\frac{\pi}{3} + 2k\pi, \text{ for some integer } k.$$

Taking the cube root gives $x = \sqrt[3]{\frac{\pi}{3} + 2k\pi}$ or $x = \sqrt[3]{-\frac{\pi}{3} + 2k\pi}$. You try it:

- Give the general solution to $\sin(x) = \frac{1}{2}$.

- Give the general solution to $2\cos(x - 1) = \sqrt{2}$.

Circular Motion:
You are standing on a circle. If you know the radius, r, of a circle and your angle θ around the circle measured from the positive x axis. Then using trigonometry you find that the x and y coordinates of your location relative to the center of the circle are given by

$$x = r\cos(\theta) \ , \ y = r\sin(\theta).$$

In high school if you had to fill in the 'unit circle' this is the relationship you were discussing (with radius $r = 1$).
This becomes even more interesting, and applicable, when we start to talk about circular motion. Assume we start at an initial angle θ_0. So at time $t = 0$, our location would be $x = r\cos(\theta_0)$ and $y = r\sin(\theta_0)$.
Then we start to run counterclockwise at a constant speed such that we complete one lap (revolution) in 10 minutes. So our 'angular speed' is $1/10$ rev/min, which is the same as $360/10$ deg/min or the same as $2\pi/10$ rad/min. So we are rotating at $\pi/5$ rad/min. We label this $\omega = \pi/5$ rad/min and call this the angular speed.
After 1 minute our angle will be $\theta = \theta_0 + 1\pi/5$, so $x = r\cos(\theta_0 + 1\pi/5)$ and $y = r\sin(\theta_0 + 1\pi/5)$.
After 2 minute our angle will be $\theta = \theta_0 + 2\pi/5$, so $x = r\cos(\theta_0 + 2\pi/5)$ and $y = r\sin(\theta_0 + 2\pi/5)$.
After 3 minute our angle will be $\theta = \theta_0 + 3\pi/5$, so $x = r\cos(\theta_0 + 3\pi/5)$ and $y = r\sin(\theta_0 + 3\pi/5)$.
In general, after t minutes, our angle will be $\theta = \theta_0 + t\pi/5$ so $x = r\cos(\theta_0 + t\pi/5)$ and $y = r\sin(\theta_0 + t\pi/5)$. This is where we get the general formula for circular motion as $x = r\cos(\theta_0 + \omega t)$ and $y = r\sin(\theta_0 + \omega t)$ (remember these are the correct location relative to the center of the circle, that is, if you are using the center of the circle as the origin, you must appropriate shift your final answer if you aren't using the center of the circle as the origin).
You try it:

- The center of a Ferris wheel is 50 feet off the ground. The radius of the Ferris wheel is 45 feet. You get on the Ferris wheel with your sweetheart at the location directly below the center. The Ferris wheel completes one revolution every two minutes. In 75 seconds, how high will you be off the ground?