MATH 124B
Exam 1
Version 1
April 20, 2004

Name _____

Section _____

1	8	
2	15	
3	4	
4	6	
5	8	
6	9	
Total	50	

- You are allowed to use a scientific calculator and one sheet of hand-written notes.

- Check that your exam contains six questions.

- Show all your work and clearly indicate your final answer.

- If you use a trial-and-error or guess-and-check method to solve a problem when an algebraic method is available, you will not receive full credit.

- Give answers with appropriate units.

- Raise your hand if you have a question.

- You have 80 minutes to complete the exam.

- Please put your name on your sheet of notes and turn it in with the exam.

GOOD LUCK!

1. (8 points) The graph of $f(x)$ is given below.
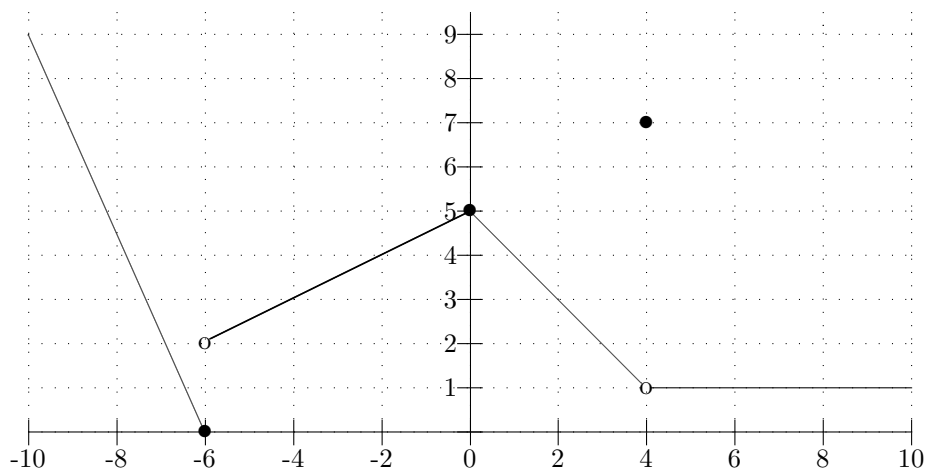
You need not show any work to answer the following questions.

(a) Evaluate each of the following. If the limit does not exist, write "DNE."

    i. $\lim\limits_{x \to -6^+} f(x)$

    ii. $\lim\limits_{x \to -6} f(x)$

    iii. $\lim\limits_{x \to 4} f(x)$

    iv. $\lim\limits_{x \to -10^+} f(x)$

    v. $\lim\limits_{x \to 2} f(x)$

(b) Is $f(x)$ continuous:

    i. at -6? circle one: YES   NO

    ii. at 0? circle one: YES   NO

    iii. at 4? circle one: YES   NO

2. (15 points) Evaluate each of the following limits. Show all work used to justify your answer. Numerical justifications using your calculator will not receive full credit. If a limit does not exist, explain why.

(a) $\displaystyle\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$

(b) $\displaystyle\lim_{x \to \infty} \frac{2x + 1}{3x^2 + 2x - 7}$

(c) $\displaystyle\lim_{x \to 2^-} \frac{x + 3}{2 - x}$

(d) $\displaystyle\lim_{x \to 4^+} \frac{x - 4}{|4 - x|}$

(e) $\displaystyle\lim_{x \to 3} \frac{x^2 + x - 12}{x^2 - 9x + 18}$

3. (4 points) Which value of c will make the following function continuous at all values of x?

$$f(x) = \begin{cases} cx - 3, & \text{if } x < 2 \\ 3 - x + 2x^2, & \text{if } x \geq 2 \end{cases}$$

4. (6 points) Let $g(x) = \dfrac{1}{x-4}$.

   (a) Use the definition of the derivative,

$$g'(a) = \lim_{h \to 0} \frac{g(a+h) - g(a)}{h},$$

   to compute $g'(a)$.

   (b) Find the equation of the line tangent to $g(x)$ at the point $\left(0, -\frac{1}{4}\right)$.

5. (8 points) Public health records indicate that t weeks after the outbreak of a certain form of influenza, approximately $Q(t) = \dfrac{20}{1 + 19e^{-1.2t}}$ thousand people had caught the disease.

   (a) How many people had the disease when it broke out?

   (b) How many people had the disease two weeks after the outbreak?

   (c) Compute $\lim\limits_{t \to \infty} Q(t)$ and explain the practical meaning of this limit.

6. (9 points) Alison likes to jog on the Burke-Gilman Trail. As a warm-up, she walks to the trail from her apartment. At $t = 0$, she begins to run. Her distance from home after t seconds is given by a function $s(t)$, measured in feet. The formula for Alison's average velocity over an interval that is h seconds long, starting at time $t = a$, is

$$v_{av} = \frac{s(a+h) - s(a)}{h} = 0.002a + 0.001h + 8.8.$$

(a) What is Alison's average velocity over the interval from $t = 120$ to $t = 150$ seconds?

(b) Find a 10-second interval over which Alison's average velocity is 11.81 feet per second.

(c) What is Alison's instantaneous velocity at $t = a$?