

1. (10 points) Don't do your work in your head, do it on the page (show me all intermediate steps you use). You do NOT have to simplify your final answer. Put a box around your final answer.

(a) Find $\frac{dy}{dx}$, if $y = \frac{7x}{5} + 6\sqrt{x^2+4}$. $= \frac{7}{5}x + 6(x^2+4)^{1/2}$

$$\boxed{\frac{dy}{dx} = \frac{7}{5} + 3(x^2+4)^{-1/2} \cdot 2x = \frac{7}{5} + \frac{6x}{\sqrt{x^2+4}}}$$

(b) Find $f'(x)$, if $f(x) = (4x^3-1)^6 \cdot \left(\frac{x^3}{2} + 7x\right)$. $= (4x^3-1)^6 \left(\frac{1}{2}x^3 + 7x\right)$

$$\boxed{f'(x) = \underbrace{(4x^3-1)^6}_F \underbrace{\left(\frac{3}{2}x^2 + 7\right)}_{S'} + \underbrace{6(4x^3-1)^5 \cdot 12x^2}_{F'} \underbrace{\left(\frac{1}{2}x^3 + 7x\right)}_S}$$

(c) The height of a balloon is given by: $h(t) = \frac{t^2 + 4\sqrt{t}}{t^4 + 1}$, where distance is in feet and time is in seconds. Find the instantaneous speed of the balloon at $t = 1$ second. (simplify your numbers and include the units for your final answer).

$$h'(t) = \frac{(t^4+1)(2t + 2t^{-1/2}) - (t^2+4\sqrt{t})4t^3}{(t^4+1)^2}$$

$$h'(1) = \frac{(2)(4) - (8)(4)}{(2)^2} = \frac{8-20}{4} = \frac{-12}{4}$$

$$= \boxed{-3 \text{ ft/sec}}$$

2. (6 pts) Assume $f(x) = \frac{32}{x} + 2x = 32x^{-1} + 2x$

(a) Find the second derivative $f''(x)$.

$$f'(x) = -32x^{-2} + 2$$

$$f''(x) = 64x^{-3} = \frac{64}{x^3}$$

(b) Solve to find all value(s) of x at which the slope of the tangent line to $f(x)$ is 0.

$$-32x^{-2} + 2 = 0$$

$$-\frac{32}{x^2} = -2$$

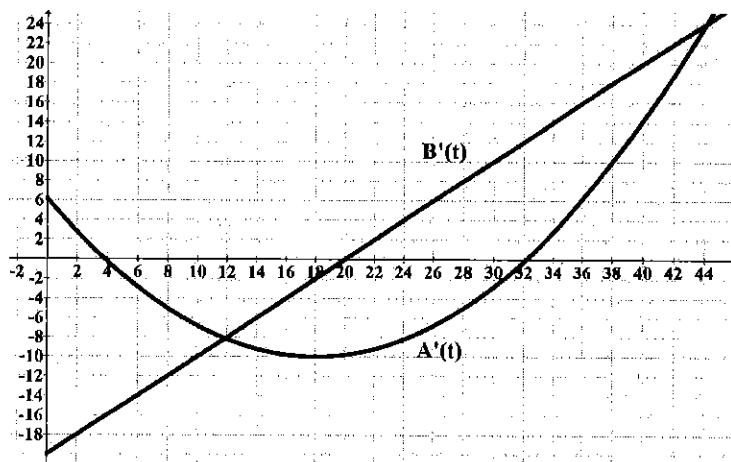
$$32 = 2x^2$$

$$16 = x^2$$

$$x = \pm 4$$

3. (10 pts)

Two balloons, A and B, start next to each other at 500 feet and are moving vertically straight up and down. Their rate of ascent graphs are shown, where t is in minutes and the rate is in feet/minute. Again, these are the graphs of the derivatives of the height functions! Use the graph to answer the following questions as accurately as possible.



(a) For each part below, circle which quantity is bigger.

i. balloon A height at $t = 12$ or balloon B height at $t = 12$ or They are equal.

ii. balloon A height at $t = 0$ or balloon A height at $t = 1$ or They are equal.

(b) Find the longest interval over which balloon A is falling and balloon B is rising.

A' negative B' positive

ANSWER: $t = 20$ min to $t = 32$ min

(c) Give all times when $A(t)$, the height graph for balloon A, has a horizontal tangent.

$$A'(t) = 0$$

ANSWER: $t = 4, 32$ min

4. (11 pts) Let $f(x) = 3x^2 - 4x + 1$.

(a) Write out and *expand and completely simplify* the formula for

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{[3(x+h)^2 - 4(x+h) + 1] - [3x^2 - 4x + 1]}{h}$$

$$\frac{3(x^2 + 2xh + h^2) - 4x - 4h + 1 - 3x^2 + 4x - 1}{h}$$

$$\frac{\cancel{3x^2} + 6xh + 3h^2 - 4h - \cancel{3x^2}}{h}$$

$$= 6x + 3h - 4$$

ANSWER: $\frac{f(x+h) - f(x)}{h} = \boxed{6x + 3h - 4}$

(b) Find the slope of the secant line to $f(x)$ from $x = 2$ to $x = 5$.

TWO WAYS

1 $x = 2, h = 3$ ABOVE $\Rightarrow 6(2) + 3(3) - 4 = 12 + 9 - 4 = \boxed{17}$

OR

2 $\frac{f(5) - f(2)}{5 - 2} = \frac{(3(5)^2 - 4(5) + 1) - (3(2)^2 - 4(2) + 1)}{3} = \frac{56 - 5}{3} = \frac{51}{3} = \boxed{17}$

ANSWER: secant slope = $\boxed{17}$

(c) Find the equation for the tangent line to $f(x)$ at $x = 2$.

$h \rightarrow 0 \Rightarrow f'(x) = 6x - 4$ OR USE SHORTCUT RULE)

HEIGHT = $f(2) = 3(2)^2 - 4(2) + 1 = 12 - 8 + 1 = 5$

SLOPE = $f'(2) = 6(2) - 4 = 12 - 4 = 8$

$y = 8(x - 2) + 5 = 8x - 16 + 5 = 8x - 11$

ANSWER: $y = \boxed{8(x - 2) + 5}$

5. (13 pts) You are in charge of marketing a new electronic gadget. From analyzing the market, you find the demand curve is

$$p = 55 - 2x,$$

where p is price (in dollars per item) and x is in **hundred** items. From the manufacturer, the total cost function is given by

$$TC(x) = 15 + 7x - 2x^2 + x^3,$$

where x is in hundred items and $TC(x)$ is in hundred dollars. Keep final answers accurate to two digits after the decimal (*i.e.* to the nearest item or nearest dollar).

- (a) Find formulas for TR , MR and MC .

$$TR(x) = 55x - 2x^2$$

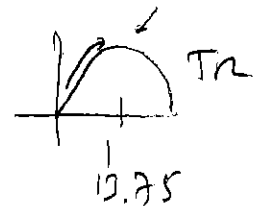
$$MR(x) = 55 - 4x$$

$$MC(x) = 7 - 4x + 3x^2$$

- (b) Find the largest interval on which total revenue is increasing.

$$MR(x) = 55 - 4x \stackrel{?}{=} 0$$

$$\Rightarrow x = \frac{55}{4} = 13.75$$



ANSWER: from $x = \underline{0}$ to $x = \underline{13.75}$ hundred items

- (c) Find the quantity at which marginal cost is lowest.

$$MC \text{ lowest} \Rightarrow MC'(x) = 0$$

$$-4 + 6x \stackrel{?}{=} 0 \Rightarrow x = \frac{4}{6} = 0.667$$

ANSWER: $x = \underline{0.67}$ hundred items

- (d) What selling price should you use to maximize profit? (Hint: First, find the quantity that maximizes profit).

$$MR(x) = MC(x)$$

$$x = 4 \Rightarrow$$

$$55 - 4x \stackrel{?}{=} 7 - 4x + 3x^2$$

$$\Rightarrow 48 = 3x^2$$

$$\Rightarrow 16 = x^2 \Rightarrow x = \pm 4$$

$$p = 55 - 2(4) = 55 - 8 = 47$$

ANSWER: selling price = 47 dollars/item