12.1 and 12.3 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

Motivation: Since we now know how to go from an 'original' function to it's derivative, we ask the natural question of how can we go from the derivative back to the original function. This is an important question in business. Take situations where marginal information (MR, MC, or MP) is known, but total information is not given. How do we translate information about MR, MC, and MP, back to TR, TC, and P? In general, if we know information about rates (speeds, rate of ascent, rate of flow, etc), how can we get total information about the original quantity (distance, height, total amount, etc).

Terminology/Facts:

- A $f(x)$ and $g(x)$ are two functions such that $f'(x) = g(x)$. Then we say that $g(x)$ is the *derivative* of $f(x)$ and we say that $f(x)$ is an *antiderivative* of $g(x)$.
- A given function will have many antiderivatives.
  For example, take the derivative of $f(x) = x^2 + 1$, $g(x) = x^2 + 500$, and $h(x) = x^2 - 124.12$.
  In all three cases, you get $f'(x) = g'(x) = h'(x) = 2x$. So all three of these functions are antiderivatives of $2x$.
  But there is an important theorem that says all antiderivatives will only differ by a constant.
  So we say that the *general antiderivative* of $2x$ is $x^2 + C$ for some constant C.
  We call C the *constant of integration* and we don't know what value it takes on unless we are given additional information about the original function (more about this in 12.4).
- The standard notation for finding general antiderivatives involves the integral sign. We write

  $$\int 2x\, dx = x^2 + C.$$

  As a short hand for: "The general antiderivative of $2x$ is $x^2 + C$".
  The $\int$ is called the integral sign.
  The dx tells us which variable we are using.
  For now you can just think of $\int ???dx$ as parentheses that say "find the general antiderivative of what is in here". In chapter 13, we will clarify more how to interpret this notation.

IMPORTANT NOTE: Remember that we can differentiate almost anything we want by using combinations of our rules. **We don't have good general ways to find antiderivatives!** The only antiderivatives we will do in this class are cases that come directly from the basic derivative rules and are easy to reverse. If you want to learn more integration techniques you have to take our full Math 124/125 calculus sequence. But even at the end of that course, there are many problems we still can't do in an exact way. So finding antiderivatives is general harder than finding derivatives. But in this class it won't be hard because we will only give you problems that can be done with the basic set of rules.

We will be able to do every problem in this course by expanding, using the sum/coefficients rules, and using the four rules below:

NAME	RULE	EXAMPLE/COMMENTS
Constant Rule	$\int a\,dx = ax + C$	$\int 7\,dx = 7x + C$
Power Rule	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$	$\int x^3\,dx = \frac{1}{4}x^4 + C$ (works for $n \neq -1$)
Logarithm Rule	$\int \frac{1}{x}\,dx = \ln(x) + C$	(works for positive x)
Exponential Rule	$\int e^{nx}\,dx = \frac{1}{n}e^{nx} + C$	$\int e^{4x}\,dx = \frac{1}{4}e^{4x} + C$

It's good to know some allowable manipulation rules as well. The sum and coefficient rules still hold, so we are allowed to do manipulations like the following:

NAME	RULE	EXAMPLE/COMMENTS
Coefficients	$\int cf(x)\,dx = c\int f(x)\,dx$	$\int 4x^2\,dx = 4\int x^2\,dx = 4 \cdot \frac{1}{3}x^3 + C$
Sums/Differences	$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$	$\int x - \frac{1}{x}\,dx = \frac{1}{2}x^2 - \ln(x) + C$

Here is a guide to integrating in this class:

1. Clean up first:
   - Rewrite the powers: At the beginning, write roots as fractional powers and write $\frac{1}{x^a} = x^{-a}$. However, if you see $\frac{1}{x}$, just leave it like that, **don't** write it as x^{-1}.
   - Expand everything out:
     If you see $x(x^2 + 5x^3)$, then expand to get $x^3 + 5x^4$.
     If you see $\frac{x-6}{x^5}$, then expand to get $x^{-4} - 6x^{-5}$.
2. Use the integration rules: For every integral in this class, if you have cleaned up the integrand correctly, then you are one step from being done! Bring along the coefficients and sums and use the rules.
3. Check your work!!! If you know how to differentiate, then you can always take the derivative of your final answer and verify that you get the same function that you started with. So you should always know your answer is correct at the end, by checking your work.

On the next page are several examples.

1. Find $\int 3 - \frac{4}{\sqrt{x}}\, dx$

   (a) Clean up: $\int 3 - 4x^{-1/2}\, dx$

   (b) Coeff and Sum/Difference come along, so our answer will look like: $3??? - 4??? + C$

   (c) Use rules: $3x - 4\frac{1}{1/2}x^{1/2} + C = 3x - 8\sqrt{x} + C$.

   (d) Check: Differentiating our final answer gives $3 - 8 \cdot \frac{1}{2}x^{-1/2} = 3 - \frac{4}{\sqrt{x}}$, so we did it right!.

2. Find $\int x^2 \left(x - \frac{5}{x^3}\right) dx$

   (a) Clean up/Expand: $\int x^3 - \frac{5}{x}\, dx = \int x^3 - 5\frac{1}{x}\, dx$

   (b) Coeff and Sum/Difference come along, so our answer will look like: $?? - 5?? + C$.

   (c) Use rules: $\frac{1}{4}x^4 - 5\ln(x) + C$.

   (d) Check: Differentiating our final answer gives $x^3 - 5\frac{1}{x}$, so we did it right!.

3. Find $\int \frac{2x - 5x^2}{x^5}\, dx$

   (a) Clean up/Expand: $\int \frac{2x}{x^5} - \frac{5x^2}{x^5}\, dx = \int 2x^{-4} - 5x^{-3}\, dx$

   (b) Coeff and Sum/Difference come along, so our answer will look like: $2?? - 5?? + C$.

   (c) Use rules: $2 \cdot \frac{1}{-3}x^{-3} - 5 \cdot \frac{1}{-2}x^{-2} + C = -\frac{2}{3x^3} + \frac{5}{2x^2} + C$.

   (d) Check: Differentiating our final answer gives $2x^{-4} - 5x^{-3}$, so we did it right!.

4. Find $\int \frac{5}{\sqrt{x^3}} - 4e^{2x} + \frac{1}{e^{5x}}\, dx$

   (a) Clean up/Expand: $\int 5x^{-3/2} - 4e^{2x} + e^{-5x}\, dx$

   (b) Coeff and Sum/Difference come along, so our answer will look like: $5?? - 4??+?? + C$.

   (c) Use rules: $5 \cdot \frac{1}{-1/2}x^{-1/2} - 4 \cdot \frac{1}{2}e^{2x} + \frac{1}{-5}e^{-5x} + C = -10x^{-1/2} - 2e^{2x} - \frac{1}{5}e^{-5x} + C$.

   (d) Check: Differentiating our final answer gives $5x^{-3/2} - 4e^{2x} + e^{-5x}$, so we did it right!.