11.1 and 11.2 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

11.1 and 11.2: Exponential and Logarithmic Derivative Rules

We added our last two derivative rules in these sections:

$$\left[e^{f(x)}\right]' = e^{f(x)} \cdot f'(x) \quad \text{and} \quad [\ln(f(x))]' = \frac{1}{f(x)} \cdot f'(x)$$

This includes the special important cases:

$$[e^x]' = e^x \quad \text{and} \quad [\ln(x)]' = \frac{1}{x}$$

Here is a reminder of how to differentiate with these two rules added to the process (You already saw this process in my 9.5-9.8 review sheets):

1. Simplify and Rewrite powers (write roots are fractional exponents and use negative exponents). The goal here is make your starting expression easier to work with.
2. Sums and Coefficients come along for the ride. Included in this steps you should identity what is a coefficent (For example, the expression $\frac{4\ln(x^2+1)}{7}$ should be rewritten as $\frac{4}{7}\ln(x^2+1)$, so that you can see that $\frac{4}{7}$ is just a coefficient).
3. Identify the form:
   (a) Product Rule? If in the form $First \cdot Second = F \cdot S$, then use $(F \cdot S)' = F \cdot S' + S \cdot F'$.
   (b) Quotient Rule? If in the form $\frac{Numerator}{Denominator} = \frac{N}{D}$, then use $\left(\frac{N}{D}\right)' = \frac{D \cdot N' - N \cdot D'}{D^2}$.
   (c) Chain Rule? If in the form $(f(x))^n$, $e^{f(x)}$ or $\ln(f(x))$, then use the appropriate chain rule ($nf(x)^{n-1} \cdot f'(x)$, $e^{f(x)} \cdot f'(x)$, or $\frac{1}{f(x)} \cdot f'(x)$, respectively).
4. In the process of using these rules, you may have to do other derivatives. In which case start this analysis over on those sub derivatives and put the results back in the correct location in the previous rule. Be organized!

You have already seen dozens and dozens of examples in lecture, in the homework, and posted online, but on the next page are three more fully worked out examples to illustrate the ideas.

1. $f(x) = 5e^{\sqrt{x}} - \ln(x^2+1)$. Find $f'(x)$.

   (a) Simplify and Rewrite Powers to get: $f(x) = 5e^{x^{1/2}} - \ln(x^2+1)$.

   (b) Sums and Coefficients come along for the ride so the final answer will look like: $f'(x) = 5???-???$.

   (c) Now we focus on how to differentiate each term. These are chain rule problems $e^{x^{1/2}}$ becomes $e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}$ and $\ln(x^2+1)$ becomes $\frac{1}{x^2+1} \cdot 2x$.

   (d) And putting this all together gives

   $$f'(x) = 5e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2} - \frac{1}{x^2+1} \cdot 2x = \frac{5e^{\sqrt{x}}}{2\sqrt{x}} - \frac{2x}{x^2+1}.$$

   You might notice that at the very end I simplified by rewriting the fractions and exponents. This sure makes the expressions easier to work with and look at, but is not required if you are just finding the derivative.

2. $g(x) = x^3e^{5x}$. Find $g'(x)$.

   (a) No powers to rewrite and no sums or coefficients to bring down.

   (b) This is a Product Rule form, so the derivative will look like: $g'(x) = x^3{\cdot}??? + e^{5x}{\cdot}???$

   (c) To fill in the last spots, we need to know the derivatives of $S = e^{5x}$ and $F = x^3$. We get $S' = e^{5x} \cdot 5$ and $F' = 3x^2$. Putting it all together gives:

   $$g'(x) = x^3 \cdot e^{5x} \cdot 5 + e^{5x} \cdot 3x^2 = 5x^3e^{5x} + 3x^2e^{5x}.$$

   Again, I rearranged at the end to make it look nice, this is not required.

3. $h(x) = \sqrt{\ln(x^2+e^x)}$. Find $h'(x)$.

   (a) Rewriting the powers gives: $h(x) = (\ln(x^2+e^x))^{1/2}$

   (b) This is a chain rule problem of the form $(?????)^{1/2}$, so the derivative will look like: $h'(x) = \frac{1}{2}(\ln(x^2+e^x))^{-1/2}{\cdot}???$

   (c) To fill in the last spots, we need to know the derivative of the inside function $\ln(x^2+e^x)$ which would be $\frac{1}{x^2+e^x} \cdot (2x+e^x)$. Putting it all together gives:

   $$h'(x) = \frac{1}{2}(\ln(x^2+e^x))^{-1/2} \cdot \frac{1}{x^2+e^x} \cdot (2x+e^x) = \frac{2x+e^x}{2(x^2+e^x)\sqrt{\ln(x^2+e^x)}}.$$

   Yet again, I rearranged at the end to make it look nice, this is not required.