10.3 and Solving Equations Overview

All the main connections and tests between functions and their first and second derivatives are on my previous review sheet for 10.1-10.3 (check it out). The only remaining topic was global maximum and global minimum.

1. The *Global Maximum* of a function over an interval is the largest output from the function. (so it is a y-value).
2. The *Global Minimum* of a function over an interval is the smallest output from the function. (again, it is a y-value).

The key observation in class was:
The global maximum and global mininum must occur at either a critical number or an endpoint!

Global Maximum/Minimum Analysis Given a function and an interval:

1. Critical Numbers: Find $f'(x)$ and solve $f'(x) = 0$.
2. Plug into the **original function**:
   (a) Plug in Critical Numbers: Take each critical number in the interval and plug them into the original function.
   (b) Plug in Endpoints: Take both endpoints and plug them into the original function.
3. Conclusions: The biggest output is the global max and the smallest output is the global minimum.

At this point you should be getting faster and faster at doing these sorts of problems and using the connections. Here is a summary of essentially ALL the questions we can ask and the synopsis of the methods you use:

1. *Question*: Find the locations at which local maximum and local mininum occur for $f(x)$.
   First Steps: Find $f'(x)$ and solve $f'(x) = 0$. Then either draw the 1st derivative number line or plug into the second derivative to classify your points.
2. *Question*: Find the intervals when $f(x)$ is increasing or decreasing.
   First Steps: Find $f'(x)$ and solve $f'(x) = 0$. Then either draw the 1st derivative number line and figure out when the derivative is positive/negative.
3. *Question*: Find the global maximum and global minimum of $f(x)$ on a given interval.
   First Steps: Find $f'(x)$ and solve $f'(x) = 0$. Then plug the critical numbers and the endpoints into the original function and see what has the biggest and smallest output.
4. *Question*: Find the points of inflection.
   First Steps: Find $f''(x)$ and solve $f''(x) = 0$. Then draw the 2nd derivative number line and figure out when the 2nd derivative is positive/negative. (The points of inflection are the places where $f''(x) = 0$ and concavity changes).
5. *Question*: Find the intervals when $f(x)$ is concave up or concave down.
   First Steps: Find $f''(x)$ and solve $f''(x) = 0$. Then draw the 2nd derivative number line and figure out when the 2nd derivative is positive/negative.