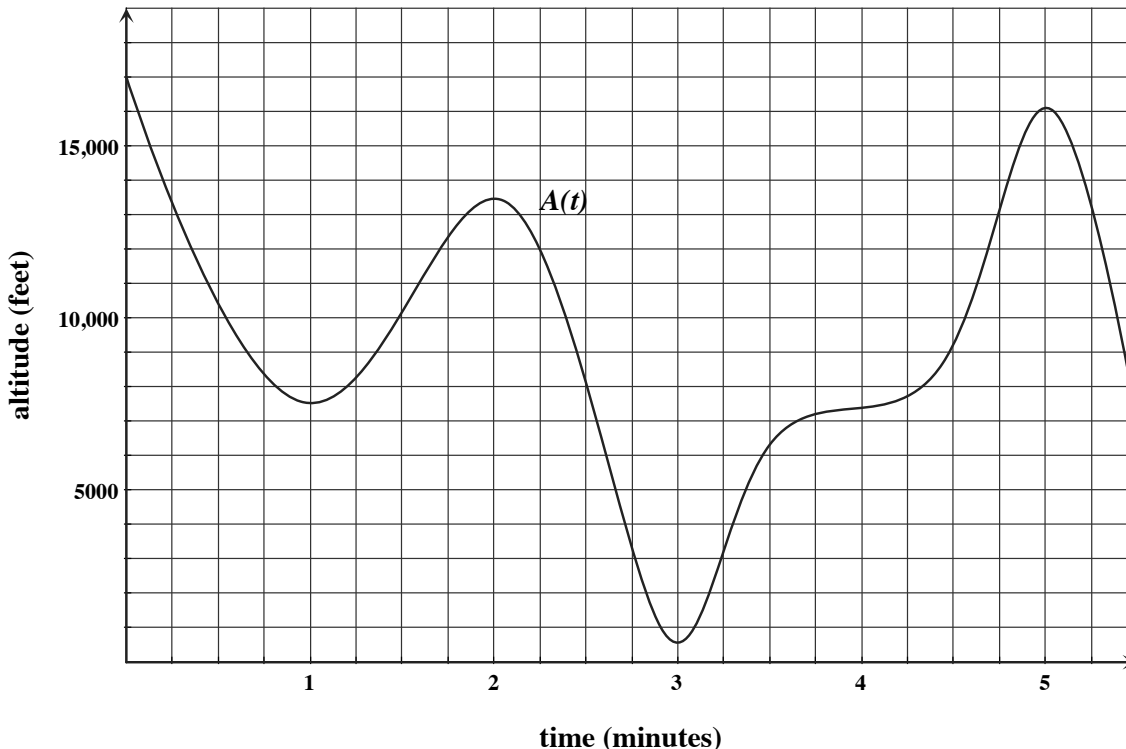


Math 112
Group Activity: Local vs. Global Optima

1. Below is the graph of altitude, $A(t)$, for a balloon that is rising and falling.



- (a) List all critical values of the function $A(t)$.
- (b) Which of these give local (relative) maxima? local minima? horizontal points of inflection?
- (c) On the interval from $t = 0$ to $t = 5.5$, the balloon is at its highest altitude at $t = 0$. We say that the **global maximum** of $A(t)$ occurs at $t = 0$. What is that altitude?
- (d) At what time does the **global minimum** of $A(t)$ (the *lowest* altitude) occur? What is the lowest altitude the balloon reaches on the interval shown?
- (e) We're now going to restrict our attention to smaller time intervals. Fill in the following chart with the values of t_{max} , the value of t that gives the global maximum altitude on the given interval, and t_{min} , the value of t that gives the global minimum altitude on the given interval.

Interval	t_{max}	t_{min}
from $t = 0$ to $t = 5.5$	0	3
from $t = 0.75$ to $t = 2.25$		
from $t = 1.75$ to $t = 2.75$		
from $t = 4.5$ to $t = 4.75$		

The previous exercise demonstrates the following fact: On the interval from $x = a$ to $x = b$, the global maximum value of a function $f(x)$ occurs either at a critical value or at one of the endpoints of the interval. That is, the global maximum value of $f(x)$ is either a local maximum value of $f(x)$ OR it is $f(a)$ or $f(b)$. Similarly, the global minimum value of $f(x)$ is either a local minimum value of $f(x)$ OR it is $f(a)$ or $f(b)$.

This gives a convenient process to find the local and global maximum and minimum values of a function $f(x)$ on the interval from $x = a$ to $x = b$.

Step 1: Compute $f'(x)$, set $f'(x) = 0$, and solve for x . This gives the critical values for $f(x)$.

Step 2: Evaluate $f(x)$ at each of the critical values **that lie between a and b** (you can ignore any critical values that are not in your interval). Also, compute $f(a)$ and $f(b)$.

Step 3: Use the information gathered in Steps 1 and 2 to SKETCH A ROUGH GRAPH of $f(x)$ on the interval from a to b . You should be able to see on your graph all local and global optima.

2. The total revenue and total cost (both in hundreds of dollars) for selling q hundred Shrubnods are given by:

$$TR(q) = -0.08q^2 + 2.35q \text{ and } TC(q) = 0.01q^3 - 0.3q^2 + 3q + 4.$$

(a) Compute the formula for profit $P(q)$ and its derivative $P'(q)$.

(b) Find all critical values of the profit function. (You may round your answers to 2 digits after the decimal.)

(c) If you produce between 10 hundred and 15 hundred Shrubnods, what production level will yield the largest profit? (Follow the three-step method given above: You've already done Step 1. Evaluate $P(q)$ at any relevant critical values and the endpoints of your interval. Use that information to sketch the graph of $P(q)$ from $q = 10$ to $q = 15$. Then you can see the optima.)

- (d) If you produce between 9 hundred and 10 hundred Shrubnods, what production level will yield the largest profit?
- (e) If you can produce any number of Shrubnods, what production level will yield the largest profit? What is the largest possible profit for producing Shrubnods? (To answer this question, sketch a rough graph of the entire profit function. Include the correct “ y ”-intercept and all critical points.)
- (f) What is the global minimum value of profit on the interval from $q = 0$ to $q = 5$ hundred Shrubnods? (NOTE: Your answer should be negative. If the smallest possible profit is negative, then its absolute value is the largest possible loss.)
- (g) If you can produce any number of Shrubnods, what production level will cause you to lose the most money? What is the largest possible loss for producing Shrubnods?