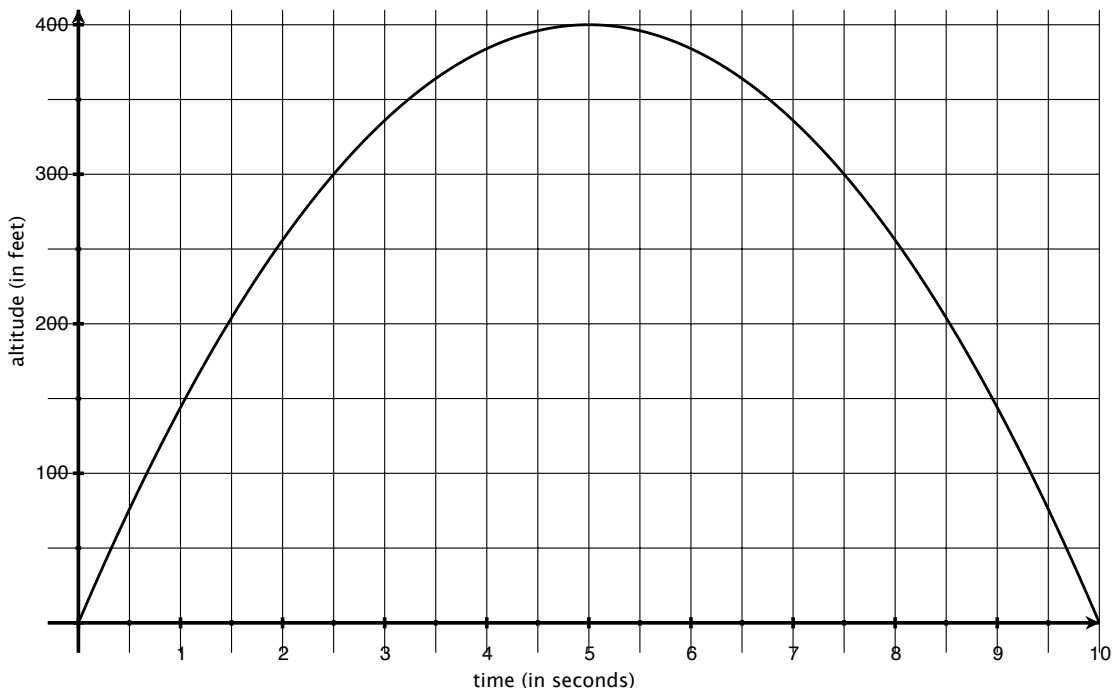


Math 112

Group Activity: The Vertical Speed of a Shell

A shell is fired straight up by a mortar. The graph below shows its altitude as a function of time.



The function describing the shell's altitude at time t will be denoted by $f(t)$, where altitude is in feet and t is in seconds.

- Use the altitude graph to approximate the following quantities:
 - the time required for the shell to reach the altitude 300 ft for the first time
 - the change in altitude from time $t = 2$ to $t = 4$
 - the average speed of the shell from time $t = 1$ to time $t = 4$

The formula for the altitude (in feet) after t seconds is given by: $f(t) = 160t - 16t^2$.

- Use this formula to compute the exact values of the quantities from question #1. Check these values against the approximations you read from the graph.
 - the time required for the shell to reach the altitude 300 ft for the first time
 - the change in altitude from time $t = 2$ to $t = 4$
 - the average speed of the shell from time $t = 1$ to time $t = 4$

Imagine that there is a tiny speedometer in the shell that records the shell's velocity just as the speedometer in your car records the car's velocity. This speedometer measures the shell's *instantaneous speed*. This is the speed the shell is traveling, not over an interval of time, but at one specific instant. We usually think of speed as distance traveled divided by time elapsed, but over a single instant, distance traveled and time elapsed are both 0 and speed is not so simple. Finding instantaneous speeds is what calculus is all about. We'll start by making some approximations.

3. We'll approximate the shell's instantaneous speed at $t = 2$ by finding the shell's average speed over a tiny time interval (an interval of length 0.01 seconds). We'll do this in two different ways:

(a) **Graphically:** On the graph of altitude, draw the line whose slope gives the shell's average speed from $t = 2$ to $t = 2.01$. That is, draw the line that goes through the graph of altitude at $t = 2$ and $t = 2.01$. Note that, on this scale, it's difficult to distinguish between $t = 2$ and $t = 2.01$. Do your best to draw such a line, extending it out as far as you can in both directions. Then estimate the coordinates of two points on the line and compute its slope. (Regardless of your choice of points, this gives the desired average speed. Make sure you understand why.)

(b) **Algebraically:** Use the formula for altitude to compute the shell's average speed from $t = 2$ to $t = 2.01$. That is, compute

$$\frac{f(2.01) - f(2)}{0.01}.$$

Compare with your answer from part (a). Would you agree that this is a good approximation of the shell's speed at the instant $t = 2$?

4. Use the same two methods to approximate the shell's instantaneous speed at $t = 5$, when it reaches its peak height. Instantaneous speed at $t = 5$ will be approximately equal to the average speed from $t = 5$ to $t = 5.01$.

(a) **Graphically:**

(b) **Algebraically:**

5. Use the same two methods to approximate the shell's instantaneous speed at $t = 0$, the instant it was launched. Instantaneous speed at $t = 0$ will be approximately equal to the average speed from $t = 0$ to $t = 0.01$.

(a) **Graphically:**

(b) **Algebraically:**

6. In #3, you approximated the shell's speed at $t = 2$ by computing the average speed over the interval from $t = 2$ to $t = 2.01$. This is a decent approximation of the speed at the instant $t = 2$ but we can get even better approximations by using even shorter time intervals.

(a) Use the algebraic method to compute the average speed of the shell over the interval from $t = 2$ to $t = 2.001$.

(b) Use the algebraic method to compute the average speed of the shell over the interval from $t = 2$ to $t = 2.0001$.

(c) What would you guess is the exact value of the instantaneous speed of the shell at $t = 2$?

(d) Describe how you could interpret the instantaneous speed of the shell at $t = 2$ on the graph of altitude as the slope of a line.