

1. (10 points) Show your intermediate work in finding your derivatives wherever possible. You do not have to simplify your derivatives.

(a) If $f(x) = \frac{15x^4}{2} - 4x + \sqrt{x^2 + 3}$, what is $f'(1)$? (i.e. find the derivative, then plug in $x = 1$)

STEP 0: COEF. & REWRITE POWER $f(x) = \frac{15}{2} x^4 - 4x + (x^2 + 3)^{\frac{1}{2}}$

STEP 1: $f'(x) = \frac{15}{2} \cdot 4x^3 - 4 \cdot 1 + \frac{1}{2}(x^2 + 3)^{-\frac{1}{2}} \cdot (2x)$

$f'(x) = 30x^3 - 4 + (x^2 + 3)^{-\frac{1}{2}} \cdot x$

$f'(1) = 30(1)^3 - 4 + (1^2 + 3)^{-\frac{1}{2}} \cdot 1$

$= 30 - 4 + 4^{-\frac{1}{2}}$

$= 26 + \frac{1}{4^{\frac{1}{2}}} = 26 + \frac{1}{2}$ $f'(1) = \underline{26.5}$

(b) If $g(x) = \left(3 - \frac{8}{x^2}\right)(2x + 6 - x^3)$, what is $g'(2)$?

STEP 0: $g(x) = (3 - 8x^{-2}) \cdot (2x + 6 - x^3) = f(x) \cdot g(x)$

STEP 1: $g'(x) = \underbrace{(3 - 8x^{-2})}_F \cdot \underbrace{(2 - 3x^2)}_S + \underbrace{(16x^{-3})}_{F'} \cdot \underbrace{(2x + 6 - x^3)}_S$

$g'(2) = (3 - 8 \cdot \frac{1}{4}) \cdot (2 - 3(2)^2) + (16 \cdot (2)^{-3}) \cdot (2(2) + 6 - (2)^3)$

$= (3 - 2) \cdot (2 - 12) + (2) \cdot (4 + 6 - 8)$

$= (1)(-10) + (2)(2) = -10 + 4$ $g'(2) = \underline{-6}$

(c) Find the equation for the tangent line to $y = \frac{3x^2 - 4x + 5}{2 - 4x}$ at $x = 0$.

(Your final answer should look like $y = mx + b$).

STEP 0: ✓ STEP 1: $y = \frac{3x^2 - 4x + 5}{2 - 4x} = \text{HEIGHT}$

$y' = \frac{(2 - 4x) \cdot (6x - 4) - (3x^2 - 4x + 5)(-4)}{(2 - 4x)^2} = \text{SLOPE}$

$y' = \frac{(2)(-4) - (5)(-4)}{(2)^2} = \frac{-8 + 20}{4} = 3$

Equation for tangent line: $y = \underline{3(x - 0) + 2.5}$