1. (12 points)

5pts (a) Find the instantaneous rate of change for $f(x) = \frac{5x^2}{2} - \frac{6}{x} + 12x^4\sqrt{x^2+8}$ at $x = 1$.

$f(x) = \frac{5}{2}x^2 - 6x^{-1} + 12x^4(x^2+8)^{1/2}$

$f'(x) = 5x + 6x^{-2} + 48x^3(x^2+8)^{1/2} + \underbrace{12x^4 \cdot \tfrac{1}{2}(x^2+8)^{-\frac{1}{2}} \cdot 2x}_{\frac{12x^5}{\sqrt{x^2+8}}}$

$f'(1) = 5 + 6 + 48\sqrt{9} + \frac{12}{\sqrt{9}}$

$= 11 + 48 \cdot 3 + 4$

ANSWER: $f'(1) =$ 159

VERSION B: CHANGE "6" TO "8"
ANSWER: $f'(1) = 161$

(b) Let $g(x) = 3x^2 - 4$.

4 pts i. Write out and *completely simplify* the formula, in terms of h, for

$$\frac{g(x+h) - g(x)}{h}.$$

AND give the derivative of $g(x)$.

$\frac{[3(x+h)^2 - 4] - [3x^2 - 4]}{h}$

$\frac{3(x^2 + 2xh + h^2) \cancel{-4} - 3x^2 \cancel{+4}}{h}$

$\frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h} = 6x + 3h$

VERSION B:
$g(x) = 3x^2 - 8$
NO CHANGE TO THIS ANSWER

ANSWER: $\frac{g(x+h)-g(x)}{h} =$ $6x + 3h$ $\qquad g'(x) =$ $6x$

4 pts ii. Find the equation for the tangent line to $g(x)$ at $x = 2$.

$y = m(x - x_0) + y_0$

2

$g(2) = 3(2)^2 - 4 = 8$

$g'(2) = 6(2) = 12$

VERSION B:
$g(2) = 4$

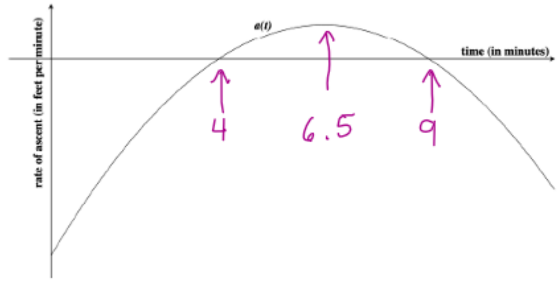
$y = 12(x-2) + 4$
$= 12x - 20$

$y = 12(x-2) + 8$

ANSWER: $y = 12x - 16$

2. At $t = 0$, Balloon A is 250 feet above the ground. Its **rate of ascent** (in feet per minute) at t minutes is

$$a(t) = -0.5t^2 + 6.5t - 18.$$



Recall, $a(t)$ is the **derivative** of the height function, so if $A(t)$ is the height of Balloon A, then $A'(t) = a(t)$.

5pts (a) Give the longest interval over which the Balloon A is rising.

A rising $\Leftrightarrow$ A' positive

NEED TO FIND WHEN $a(t) \geq 0$

$-0.5t^2 + 6.5t - 18 = 0$

$\Rightarrow t = \dfrac{-6.5 \pm \sqrt{6.5^2 - 4(-0.5)(-18)}}{2(-0.5)}$

$= \dfrac{-6.5 \pm \sqrt{6.25}}{-1}$

$= \dfrac{-6.5 \pm 2.5}{-1}$

$t = 4$ & $t = 9$

ANSWER: $t =$ 4 to $t =$ 9

4pts (b) Give the time when Balloon A is rising the fastest?

rising fastest $\Leftrightarrow$ $a(t)$ maximum $\Leftrightarrow$ $a'(t) = 0$

$a'(t) = -t + 6.5 \overset{?}{=} 0$

$\Rightarrow t = 6.5$

ANSWER: $t =$ 6.5

4pts (c) The **height** (in feet) of a second balloon, Balloon B at t minutes is given by

$$B(t) = -1.5t^2 + 24t + 250.$$

21 VERSION B

$\Rightarrow -3t + 21 = 0$

$t = 7$

$\Rightarrow a(7) = \boxed{3}$

How fast is Balloon A moving when Balloon B is at its highest altitude?

B max height $\Leftrightarrow$ $B'(t) = 0$ $\Leftrightarrow$ $-3t + 24 = 0$

$t = 8$

Rate of ascent For Balloon A $= a(8) = -0.5(8)^2 + 6.5(8) - 18 = 2$

ANSWER: Speed of Balloon A (at time when B is highest) is 2 ft/min

3. You sell Items. For sell and producing x **hundred** Items, you are given:

12

VERSION B

Demand Curve (*i.e.* price):	$p = 10 - x$	dollars/Item
Total Cost:	$TC(x) = \frac{x^3}{12} + x^2 + x + 1$	hundred dollars.

3

(a) Recall: Average cost is defined by $AC(x) = \frac{TC(x)}{x}$. Find and simplify the formula for $AC(x)$ and the formula for the derivative of average cost $AC'(x)$.

3 pts

$$AC(x) = \frac{1}{12}x^2 + x + 1 + \frac{1}{x}$$

ANSWER: $AC(x) =$ $\frac{1}{12}x^2 + x + 1 + \frac{1}{x}$

ANSWER: $AC'(x) =$ $\frac{1}{6}x + 1 - x^{-2}$

(b) Approximate the change in Total Revenue from 200 Items to 201 Items. (*You can use either Math 111 or Math 112 tools, but it is much faster to use the new tools*).

3 pts

$TR(x) = 10x - x^2$

$MR(x) = 10 - 2x$

MATH 112 WAY

CHANGE IN TR FROM 200 TO 201 $\approx MR(2) = 10 - 2(2) = 6$

MATH 111 WAY

$: TR(2.01) - TR(2)$

$= (10(2.01) - (2.01)^2) - (10(2) - (2)^2) = 0.0599$

hund# ROUNDS TO SAME

VERSION B 8

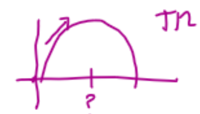
ANSWER: 6 dollars

(c) Give the longest interval over which $TR(x)$ is increasing.

3 pts

TR MAX $\Leftrightarrow$ MR $= 0$

$10 - 2x = 0 \Rightarrow x = 5$



$-\infty$ ALSO ACCEPTABLE

VERSION B 6

ANSWER: $x =$ 0 to $x =$ 5 hundred items

(d) At what quantity and selling price is profit maximum?
(Round final answer to two digits after the decimal)

5 pts

MR = MC

$10 - 2x = \frac{1}{4}x^2 + 2x + 1$

$0 = \frac{1}{4}x^2 + 4x - 9$

$$x = \frac{-4 \pm \sqrt{16 - 4(\frac{1}{4})(-9)}}{2(1/4)}$$

$$x = \frac{-4 \pm \sqrt{25}}{1/2}$$

$$x = \frac{-4+5}{1/2} = 2 \qquad x = \frac{-4-5}{1/2} = -18$$

ANSWER: quantity = 2 hundred items

$p = 10 - x = 10 - 2$ price = 8 dollars/Item

10 VERSION B