1. (13 pts) A company sells items. For all functions in this problem, x is in *thousands* of items.

The selling **price** per item is $p = 46 - 8x$ dollars/item.

The total cost, $TC(x)$, is $TC(x) = 5x + 12$ thousand dollars.

Round all final answers to three digits after the decimal.

(a) (3 pts) Find and simplify the formulas for fixed cost, FC, variable cost, $VC(x)$, and marginal cost, $AC(x)$.

$FC =$ 12 thousand dollars

$$\frac{(5(x+0.001)+12)-(5x+12)}{0.001} = 5$$

$VC(x) =$ $5x$ thousand dollars

ALSO $\longrightarrow$ slope of TC $\rightarrow$ $MC(x) =$ 5 dollars/item

(b) (2 pts) Find and simplify the formula for profit, $P(x)$.

$$TR(x) - TC(x) = (46x - 8x^2) - (5x + 12) = 46x - 8x^2 - 5x - 12$$

$P(x) =$ $-8x^2 + 41x - 12$ thousand dollars

(c) (4 pts) Find the quantity where average cost, $AC(x)$, is 10 dollars per item.

$$AC(x) = \frac{5x+12}{x} \stackrel{?}{=} 10$$

$$\Rightarrow \quad 5x + 12 = 10x$$

$$12 = 5x$$

$$x = \frac{12}{5} = 2.4$$

$x =$ 2.4 thousand items

(d) (4 pts) Find the quantity and selling price that correspond to the maximum total revenue.

$$TR(x) = 46x - 8x^2$$

$$x = \frac{-46}{2(-8)} = \frac{46}{16} = 2.875 \text{ items}$$

$$p = 46 - 8x = 46 - 8(2.875) = 23$$

$x =$ 2.875 thousand items

$p =$ 23 dollars per item

2. (16 points) Consider a different company. In this problem, x is in *hundred* of items.

Total revenue is $TR(x) = 80x - 2x^2$ hundred dollars.

Average variable cost is $AVC(x) = 0.4x^2 - 8x + 76$ dollars per item.
Fixed costs are \$2300 (which is 23 hundred dollars).

Round all final answers to two digits after the decimal.

(a) (2 pts) Give the formula for Total Cost, $TC(x)$ and price per item, p.

$TC(x) =$ $0.4x^3 - 8x^2 + 76x + 23$ hundred dollars

price per item $= p =$ $80 - 2x$ dollars per item

(b) (2 pt) What is the profit if you produce and sell 200 items?

$P(2) = (80(2) - 2(2)^2) - (0.4(2)^3 - 8(2)^2 + 76(2) + 23) = 152 - 146.2 =$

$P(1) =$ 5.8 hundred dollars

(c) (4 pts) Find the shutdown price (SDP).

LOWEST y-VALUE OF AVC ∪

$x = \frac{-b}{(2a)} = \frac{-(-8)}{2(0.4)} = \frac{8}{0.8} = 10$

$AVC(10) = 0.4(10)^2 - 8(10) + 76 = 36$

36 dollars/item

(d) (4 pts) Find the largest interval of x-values where $TR(x)$ is greater than or equal to 600 hundred dollars.

$80x - 2x^2 = 600$

$\Rightarrow\ 0 = 2x^2 - 80x + 600$

$x = \frac{80 \pm \sqrt{6400 - 4(2)(600)}}{2(2)}$

$= \frac{80 \pm \sqrt{1600}}{4} = \frac{80 \pm 40}{4}$

$= \frac{120}{4}$ OR $\frac{40}{4}$

from $x =$ 10 to $x =$ 30 hundred items

(e) (4 pts) Find and completely simplify $MR(x) = \dfrac{TR(x+0.01) - TR(x)}{0.01}$.

$\frac{[80(x+0.01) - 2(x+0.01)^2] - [80x - 2x^2]}{0.01}$

$\frac{\cancel{80x} + 0.8 - 2(x^2 + 0.02x + 0.0001) - \cancel{80x} + 2x^2}{0.01}$

$\frac{0.8 - \cancel{2x^2} - 0.04x - 0.0002 + \cancel{2x^2}}{0.01} = 80 - 4x - 0.02$

$MR(x) =$ $79.98 - 4x$ dollars/item

3. (11 pts)

(a) The demand function for a product is given by $173 - 4p = q$, where p is the price per item, in dollars/item, and q in the number of items.

The supply function for the same product is **linear**. Suppliers produce 10 items if the price is 25 dollars/item and produce 20 items if the price is 40 dollars/item.

i. (3 pts) Find linear function for the supply curve.

$$m = \frac{40-25}{20-10} = \frac{15}{10} = 1.5$$

$$p = 1.5(q-10)+25 = 1.5q - 15 + 25$$

$p =$ $1.5q + 10$

ii. (4 pts) Find the price and quantity that correspond to market equilibrium.

COMBINE $173 - 4p = q$

$$173 - 4(1.5q+10) = q$$

$$173 - 6q - 40 = q$$

$$133 = 7q$$

$$q = \frac{133}{7} = 19$$

$$p = 1.5(19)+10$$

$$= 38.5$$

$q =$ 19 items

$p =$ 38.5 dollars/item

iii. (1 pt) Does a market price of \$47 per item correspond to a shortage or surplus?

$47 > 38.5$

Circle one: Shortage or Surplus

(b) (3 pts) Solve $5 + 4e^{3x} = 15$.

Give your final answer **as a decimal**, accurate to three digits after the decimal.

$$4e^{3x} = 10$$

$$e^{3x} = 2.5$$

$$3x = \ln(2.5) \approx 0.916290732$$

$$x = \frac{\ln(2.5)}{3} \approx 0.305430244$$

$x \approx$ 0.305