Math 111

Exam 2

February 23, 2023

Name ____________________________________

Student ID #_______________ Section _______________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_______________________________

- This exam consists of a cover, a summary of business terms, three pages of questions, and a scratch sheet. If you put work on the scratch sheet and you want it to be graded, you must clearly tell us in the problem to "see last page".
- You will have 50 minutes.
- You are allowed to use a non-graphing scientific calculator, a ruler, and one 8.5 by 11 inch sheet of handwritten notes (front and back). All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam. You may not listen to headphones or earbuds during the exam.
- **You must show your work.** Clearly show steps in finding lines, solving, using the vertex or quadratic formulas as well as simplifying and arithmetic. The correct answer with no supporting work will result in no credit.
- Unless otherwise indicated, when rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- There are multiple versions, you have signed an honor statement, and cheating is a hassle for everyone involved. If we find that you give an answer that is only appropriate for the other version of the exam and there is no work to support your answer, then you will get a zero on the entire exam and your work will be submitted to the academic misconduct board. JUST DO NOT CHEAT.

GOOD LUCK!

Suppose you produce and sell Things. The following table summarizes the terms we've learned so far relating to revenue and cost. Assume you are given a graph of total cost $TC(q)$ and total revenue $TR(q)$ for producing and selling q Things.

Term	**Definition**	**Related equations and formulas**	**Graphical Interpretation**
total cost $TC(q)$	the total amount you spend to produce q Things	$TC(q) = VC(q) + FC$	—
variable cost $VC(q)$	the money you spend to produce q Things without including fixed costs	$VC(q) = TC(q) - FC$	the graph of VC has the same shape as TC and goes through the origin
fixed cost FC	the money you must spend even if you produce 0 Things; also known as *overhead*	$FC = TC(q) - VC(q)$ $FC = TC(0)$	the vertical distance between the TC and VC graphs OR the "y"-intercept of the TC graph
average cost $AC(q)$	total cost averaged over the number of Things produced	$AC(q) = \frac{TC(q)}{q}$	the slope of the diagonal line through the TC graph at q
average variable cost $AVC(q)$	variable cost averaged over the number of Things produced	$AVC(q) = \frac{VC(q)}{q}$	the slope of the diagonal line through the VC graph at q
breakeven price BEP	the smallest value of average cost	—	the slope of the least steep diagonal line that intersects the TC graph
shutdown price SDP	the smallest value of average variable cost	—	the slope of the least steep diagonal line that intersects the VC graph
marginal cost $MC(q)$ (see footnote)	the incremental rate of change in TC from q to $q+1$ Things	$MC(q) = \frac{TC(q+1)-TC(q)}{1}$	the slope of the secant line through TC (or VC) at q and $q+1$
total revenue $TR(q)$	the total amount you receive when you sell q Things	—	—
average revenue $AR(q)$	total revenue averaged over the number of Things sold; also known as *price per Thing*	$AR(q) = \frac{TR(q)}{q}$	the slope of the diagonal line through the TR graph at q
marginal revenue $MR(q)$ (see footnote)	the incremental rate of change in TR from q to $q+1$ Things	$MR(q) = \frac{TR(q+1)-TR(q)}{1}$	the slope of the secant line through the TR graph at q and $q+1$
profit $P(q)$	the money you are left with after subtracting total cost from total revenue	$P(q) = TR(q) - TC(q)$	the vertical distance between TR and TC (when $TR > TC$)

NOTE: If q is measured in hundreds or thousands of Things, the definitions, formulas, and graphical interpretations of marginal revenue and marginal cost must be adjusted appropriately.

1. (13 pts) A company sells items. For all functions in this problem, x is in *thousands* of items.

   The selling **price** per item is $p = 46 - 8x$ dollars/item.

   The total cost, $TC(x)$, is $TC(x) = 5x + 12$ thousand dollars.

   Round all final answers to three digits after the decimal.

   (a) (3 pts) Find and simplify the formulas for fixed cost, FC, variable cost, $VC(x)$, and marginal cost, $MC(x)$.

   $FC =$ ____________________ thousand dollars

   $VC(x) =$ ____________________ thousand dollars

   $MC(x) =$ ____________________ dollars/item

   (b) (2 pts) Find and simplify the formula for profit, $P(x)$.

   $P(x) =$ ____________________ thousand dollars

   (c) (4 pts) Find the quantity where average cost, $AC(x)$, is 10 dollars per item.

   $x =$ ____________________ thousand items

   (d) (4 pts) Find the quantity and selling price that correspond to the maximum total revenue.

   $x =$ ____________________ thousand items

   $p =$ ____________________ dollars per item

2. (16 points) Consider a different company. In this problem, x is in *hundred* of items.

   Total revenue is $TR(x) = 80x - 2x^2$ hundred dollars.

   Average variable cost is $AVC(x) = 0.4x^2 - 8x + 76$ dollars per item.
   Fixed costs are \$2300 (which is 23 hundred dollars).

   Round all final answers to two digits after the decimal.

   (a) (2 pts) Give the formula for Total Cost, $TC(x)$ and price per item, p.

   $TC(x) =$ ______________________________ hundred dollars

   price per item $= p =$ ______________________________ dollars per item

   (b) (2 pt) What is the profit if you produce and sell 200 items?

   $P(2) =$ ____________________ hundred dollars

   (c) (4 pts) Find the shutdown price (SDP).

   ____________________ dollars/item

   (d) (4 pts) Find the largest interval of x-values where $TR(x)$ is greater than or equal to 600 hundred dollars.

   from $x =$ _______________ to $x =$ _______________ hundred items

   (e) (4 pts) Find and completely simplify $MR(x) = \dfrac{TR(x+0.01) - TR(x)}{0.01}$.

   $MR(x) =$ _________________________ dollars/item

3. (11 pts)

(a) The demand function for a product is given by $173 - 4p = q$, where p is the price per item, in dollars/item, and q in the number of items.

The supply function for the same product is **linear**. Suppliers produce 10 items if the price is 25 dollars/item and produce 20 items if the price is 40 dollars/item.

i. (3 pts) Find linear function for the supply curve.

$p =$ ____________________

ii. (4 pts) Find the price and quantity that correspond to market equilibrium.

$q =$ ____________ items

$p =$ __________ dollars/item

iii. (1 pt) Does a market price of \$47 per item correspond to a shortage or surplus?

Circle one: Shortage or Surplus

(b) (3 pts) Solve $5 + 4e^{3x} = 15$.

Give your final answer **as a decimal**, accurate to three digits after the decimal.

$x \approx$ ____________

You may use this page for scratch-work or extra room.
All work on this page will be ignored unless you write and circle "see last page" on the problem and you label your work.