MATH 111
Final Exam
Winter 2022

Name _____

Student ID #_____ Section _____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- Check that your exam contains **6** problems.
- You are allowed to use a non-graphing scientific calculator, a ruler, and one 8.5 by 11 inch sheet (front and back) of hand-written notes. All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- **You must show your work.** Clearly label lines and points that you are using and show all calculations. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Unless otherwise indicated, when rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the last page and **indicate that you have done so**.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.

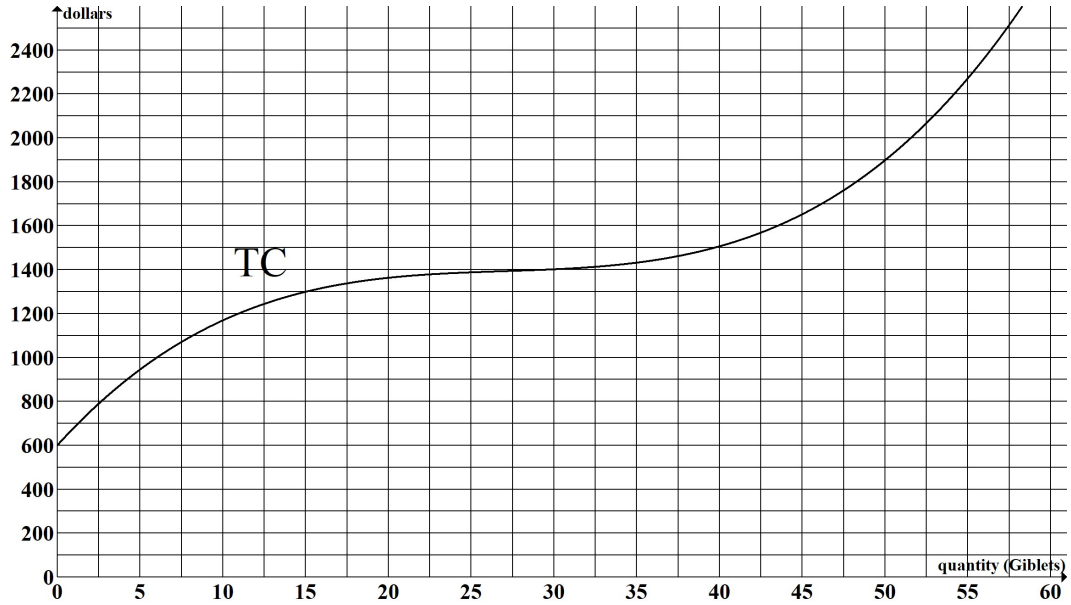
GOOD LUCK!

Suppose you produce and sell Things. The following table summarizes the terms we've learned so far relating to revenue and cost. Assume you are given a graph of total cost $TC(q)$ and total revenue $TR(q)$ for producing and selling q Things.

Term	**Definition**	**Related equations and formulas**	**Graphical Interpretation**
total cost $TC(q)$	the total amount you spend to produce q Things	$TC(q) = VC(q) + FC$	—
variable cost $VC(q)$	the money you spend to produce q Things without including fixed costs	$VC(q) = TC(q) - FC$	the graph of VC has the same shape as TC and goes through the origin
fixed cost FC	the money you must spend even if you produce 0 Things; also known as *overhead*	$FC = TC(q) - VC(q)$ $FC = TC(0)$	the vertical distance between the TC and VC graphs OR the "y"-intercept of the TC graph
average cost $AC(q)$	total cost averaged over the number of Things produced	$AC(q) = \frac{TC(q)}{q}$	the slope of the diagonal line through the TC graph at q
average variable cost $AVC(q)$	variable cost averaged over the number of Things produced	$AVC(q) = \frac{VC(q)}{q}$	the slope of the diagonal line through the VC graph at q
breakeven price BEP	the smallest value of average cost	—	the slope of the least steep diagonal line that intersects the TC graph
shutdown price SDP	the smallest value of average variable cost	—	the slope of the least steep diagonal line that intersects the VC graph
marginal cost $MC(q)$ (see footnote)	the incremental rate of change in TC from q to $q+1$ Things	$MC(q) = \frac{TC(q+1)-TC(q)}{1}$	the slope of the secant line through TC (or VC) at q and $q+1$
total revenue $TR(q)$	the total amount you receive when you sell q Things	—	—
average revenue $AR(q)$	total revenue averaged over the number of Things sold; also known as *price per Thing*	$AR(q) = \frac{TR(q)}{q}$	the slope of the diagonal line through the TR graph at q
marginal revenue $MR(q)$ (see footnote)	the incremental rate of change in TR from q to $q+1$ Things	$MR(q) = \frac{TR(q+1)-TR(q)}{1}$	the slope of the secant line through the TR graph at q and $q+1$
profit $P(q)$	the money you are left with after subtracting total cost from total revenue	$P(q) = TR(q) - TC(q)$	the vertical distance between TR and TC (when $TR > TC$)

NOTE: If q is measured in hundreds or thousands of Things, the definitions, formulas, and graphical interpretations of marginal revenue and marginal cost must be adjusted appropriately.

1. (20 points) The graph of **total cost** for producing Giblets are given. The x-axis is in Giblets and the y-axis in in dollars.



Make sure to **read the description above the graph** before you do the problems! Show and label your work in the graph. Round your estimates to the nearest cent or nearest Giblet.

(a) What is the value of fixed costs?

$FC =$ _____ dollars

(b) Find the **Break Even Price** (BEP).

$BEP =$ _____ dollars per Giblet

(c) Find the **average variable cost** at $q = 15$ Giblets.

$AVC(15) =$ _____ dollars per Giblet

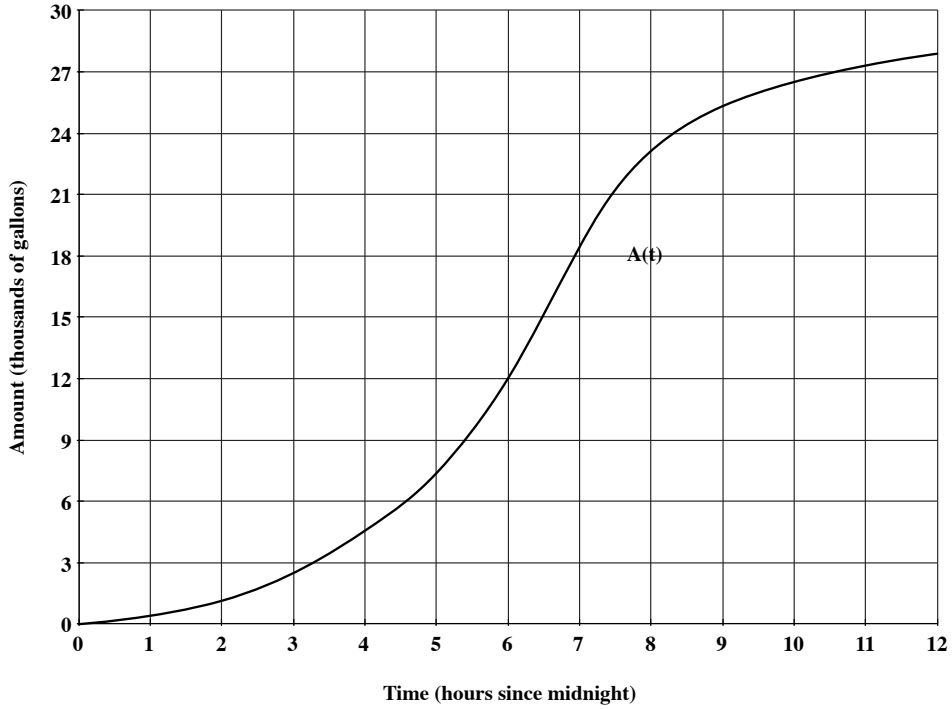
(d) Give the longest interval of quantities over which marginal cost is **at most** 20 dollars per tablet.

from $q =$ _____ to $q =$ _____ Giblets

(e) Suppose the market price is \$45.00 per Giblet. Find the quantity that maximizes profit and give the value of maximum profit.

$q =$ _____ Giblets and Profit = _____ dollars

2. (16 points) The graph below gives the amount of water, $A(t)$, that flows out of a reservoir over a 12-hour period beginning at midnight. The amount, $A(t)$, is in thousands of gallons and the time t is in hours after midnight.



Show and label your work in the graph.

(a) Find a value of t, larger than 6, such that $\dfrac{A(t) - A(6)}{t - 6} = 4.2$.

$t =$ _____ hours

(b) Find the time when the overall rate of flow out of the reservoir is largest.

$t =$ _____ hours

(c) During how many one-hour intervals is water flowing out at an average rate of 1.8 thousand gallons per hour?

number of one-hour intervals with average rates of 1.8 (Circle one): 0 1 2 3 4 5

(d) Suppose water flows into the reservoir at a constant rate of 1.8 thousand gallons per hour. What is the smallest amount of water needed in the reservoir at midnight so that the reservoir never has a shortage in this 12-hour period?

_____ thousand gallons

3. (18 points) You sell Things.
You are given that the total cost for selling x hundred Things is $TC(x) = 12x + 4000$ hundred dollars.
Also, you are told that the selling price per Thing is $p = -3x + 600$ dollars/Thing where x is in hundred Things.

(a) Give the formula for total revenue, $TR(x)$.

$TR(x) =$ _____

(b) Recall in this case that $MR(x) = \frac{TR(x+0.01)-TR(x)}{0.01}$. Find and completely simplify the formula for Marginal Revenue.

$MR(x) =$ _____ dollars/Thing

(c) Find the largest interval over which Total Revenue is greater than or equal to \$3000.
(Round answers to two digits after the decimal)

$x =$ _____ to $x =$ _____ hundred Things

(d) Find the quantity and selling price which correspond to maximum profit.

Quantity: $x =$ _____ hundred Things

Selling Price: $p =$ _____ dollars/Thing

4. (16 pts) The average variable cost of producing x **thousand** items is given by

$$AVC(x) = 0.01x^2 - 0.7x + 80 \quad \text{and} \quad MC(x) = 0.03x^2 - 1.4x + 80,$$

where $AVC(x)$ and $MC(x)$ are in dollars/item.

In addition, the selling price per item is a constant $p = 86$ dollars/item.

(a) Give the formulas/values for all the following:

i. Variable Cost: $VC(x) =$ _____ thousand dollars

ii. Total Revenue: $TR(x) =$ _____ thousand dollars

iii. Marginal Revenue: $MR(x) =$ _____ dollars/item

(b) Recall that the Shutdown Price (SDP) is the lowest value of $AVC(x)$. Find the Shutdown Price. (Round to the nearest cent)

SDP = _____ dollars/item

(c) Find the quantity at which profit is maximized. (Round to three digits after the decimal).

$x =$ _____ thousand items

5. (16 pts) (For all your work below, round your **final answer** to two digits after the decimal)

(a) Jill found an investment that will pay her 5% annual interest, compounded quarterly. How much must Jill invest in the account now so that she will have $10,000 in five years?

_____ dollars

(b) Molly deposits $500 into an account that pays 3% annually, compounded continuously. How long will it take for the account balance to double?

_____ years

(c) Fred has an account that pays interest compounded semi-annually. He deposited $600 initially and then 5 years later the account balance was $900. What is the interest rate?

_____ %

6. (14 pts) (Round to the nearest cent)

(a) You plan to take a big trip in four years (after college). You deposit \$100 at the beginning of every month for 4 years in an account with 6% annual interest compounded monthly. How much money will be in the account after 4 years AND how much interest do you earn?

Balance in 4 years = _____ dollars

Total interest earned = _____ dollars

(b) Your friend also plans to save for a trip in four years. They plan to make deposits at the **end** of every month for 4 years in an account with 6% annual interest compounded monthly. If your friend knows they will need \$15,000 for their trip how much money do they need to deposit at the end of every month?

_____ dollars

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