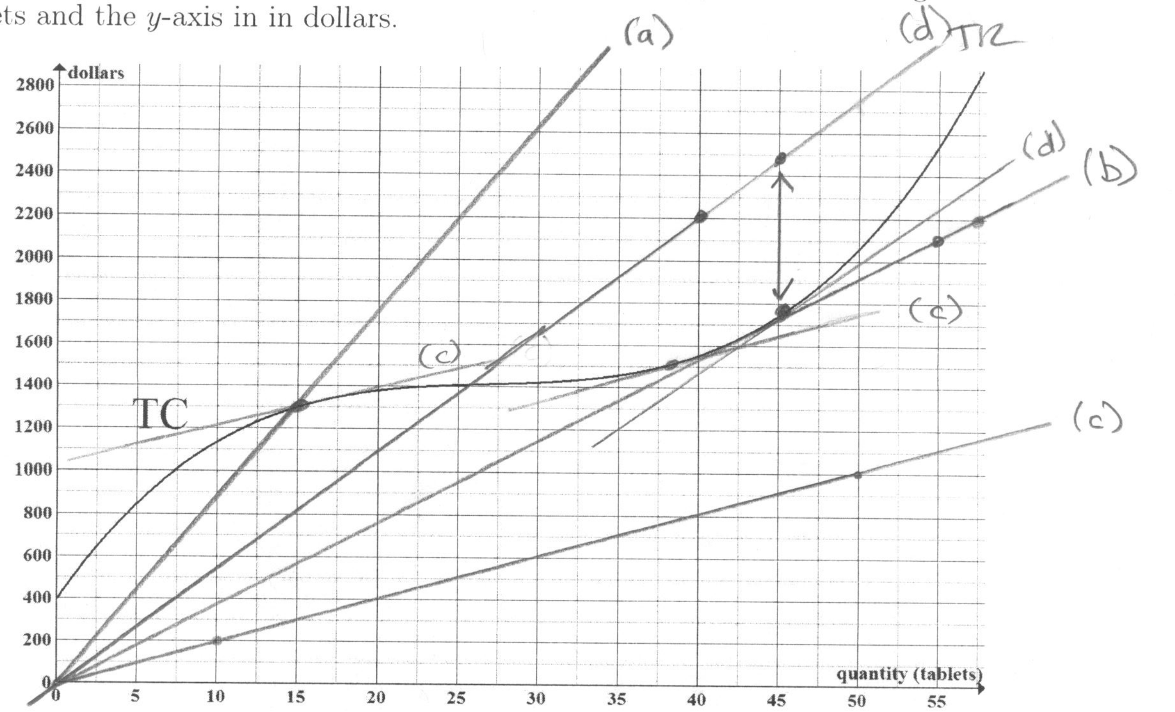
1. (15 pts) The graph of **total cost** for producing a certain type of tablet are given. The x-axis is in tablets and the y-axis in in dollars.



Show some calculations in every problem and label your work in the graph.
(At least one line should be drawn for each part!).

(a) Find the **average variable cost** and **average cost** at $q = 15$ tablets.

$AC(15) = \frac{TC(15)}{15} = \frac{1300}{15} = 86.\overline{66}$

$AVC(15) = \frac{VC(15)}{15} = \frac{1300-400}{15} = 60$

$AVC(15) =$ 60.00 dollars per tablet

$AC(15) =$ 86.67 dollars per tablet

(b) Find the **Breakeven Price** (BEP). (0,0) (55,2100)

$\text{slope} = \frac{2100-0}{55-0} = 38.\overline{18}$

[redacted] $=$ 38.18 dollars per tablet

[37.5 – 38.5]

(c) Give the longest interval of quantities over which marginal cost is less than or equal to 20 dollars per tablet.

$mc(q) \overset{?}{=} 20$ at about $q \approx 15$ AND $q \approx 38$

$mc(q) \leq 20$ between these values

from $q =$ 15 to $q =$ 38 tablets

[14 – 17] [36 – 40]

(d) Suppose the market price is \$55.00 per tablet. Find the quantity that maximizes profit and give the value of maximum profit.

(40, 2200) MATCH SLOPES $q \approx 45$ PROFIT = TR(45) – TC(45)

.55

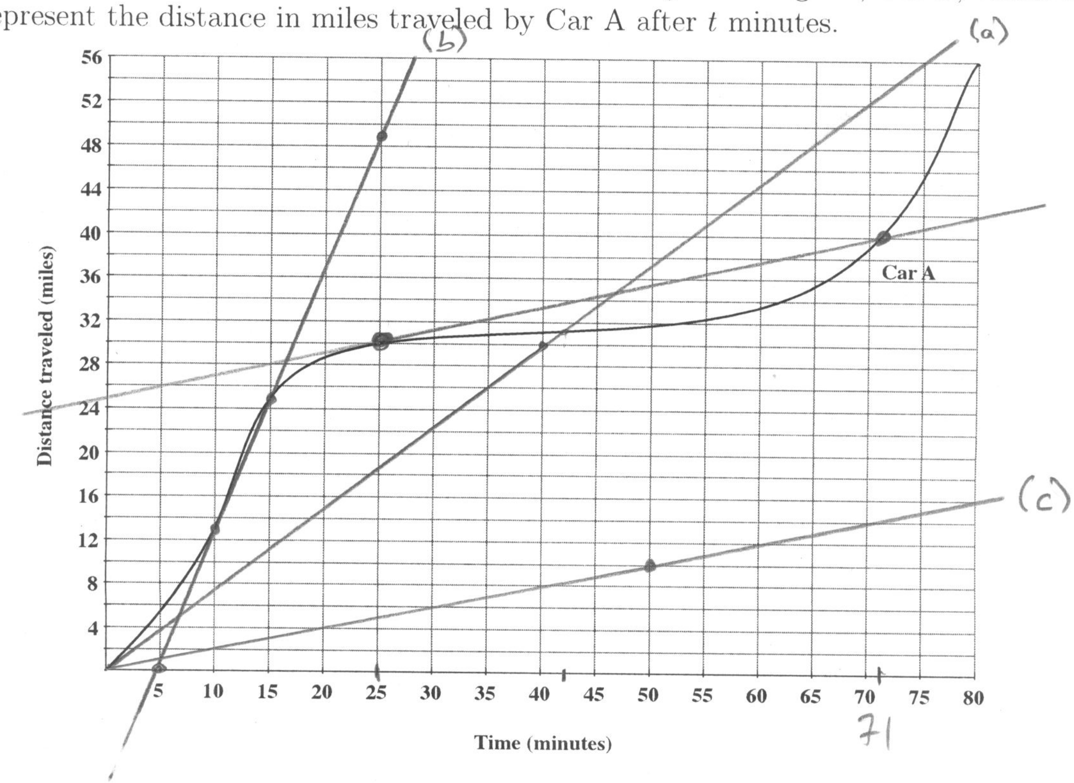
$\approx 2500 - 1750 \approx 750$

$q =$ 45 tablets and Profit $=$ 750 dollars

[44 – 48] [700 – 800]

13

2. ([redacted] pts) The graph gives total distance in miles traveled by a moving car, Car A, versus time. Let $A(t)$ represent the distance in miles traveled by Car A after t minutes.



Show some calculations in every problem and label your work in the graph.
(At least one line should be drawn for each part!)

(a) Find a time at which Car A's overall average trip speed is 0.75 miles per minute.

(40, 30)

0.75

DIAG. LINE CROSSED A(t) AT ABOUT 42

$t =$ 42 minutes

(b) Translate the following phrase into functional notation (using the function name $A(t)$ and appropriate values) AND compute the numerical answer:
Find the average speed of the car over the 5-minute interval starting at $t = 10$ minutes

(5, 0)

(25, 49)

$$\frac{49-0}{25-5} = 2.45$$

FUNCTIONAL NOTATION: $\frac{A(15) - A(10)}{15 - 10}$

"Average speed from 10 to 15" = 2.45 mpm

(c) Find a positive value of h such that $\underbrace{\frac{A(25+h) - A(25)}{h}}_{\text{SLOPE FROM}} = 0.2$ miles per minute.

(50, 10)

0.2

25 $\xleftarrow{h\text{-min}}$ $\underbrace{25+h}_{71}$

$h = 71 - 25$

$h =$ 46 minutes

14

3. ([redacted] points) Consider the two functions

$$f(x) = 5x - x^2 \quad \text{and} \quad g(x) = 3x^2 - 4x + 5.$$

(a) Find the slope of the diagonal line to $f(x)$ at $x = 3$.
(Hint: First write in functional notation, then compute).

$$\frac{f(3)}{3} = \frac{5(3) - (3)^2}{3} = \frac{15 - 9}{3} = \frac{6}{3} = 2$$

OR $\frac{f(x)}{x} = 5 - x \quad x = 3$

2

(b) Find and *completely simplify* $\dfrac{g(x+h) - g(x)}{h}$.

$$= \frac{[3(x+h)^2 - 4(x+h) + 5] - [3x^2 - 4x + 5]}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) - \cancel{4x} - 4h + \cancel{5} - 3x^2 + \cancel{4x} - \cancel{5}}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 - 4h - \cancel{3x^2}}{h}$$

$$= 6x + 3h - 4$$

$\frac{g(x+h)-g(x)}{h} =$ $6x + 3h - 4$

(c) Find all quantities at which the graphs of $f(x)$ and $g(x)$ cross.

$$5x - x^2 \stackrel{?}{=} 3x^2 - 4x + 5$$

$$0 = 4x^2 - 9x + 5$$

$$x = \frac{9 \pm \sqrt{9^2 - 4(4)(5)}}{2(4)}$$

$$x = \frac{9 \pm \sqrt{1}}{8}$$

$$x = \frac{9-1}{8} = 1$$

$$x = \frac{9+1}{8} = \frac{10}{8} = \frac{5}{4} = 1.25$$

(List solutions) $x =$ $1, 1.25$

4. (15 pts)

(a) You sell things. The selling price per thing is a constant $p = \$7.00$ per thing and your average cost to produce q things is $AC(q) = 5 + \frac{3}{q}$ dollars per thing.

i. Find formulas for total revenue and total cost in terms of q.

$TR(q) =$ $7q$ dollars

$TC(q) =$ $5q + 3$ dollars

ii. At what quantity is the average cost equal to \$5.10 per thing?

$5 + \frac{3}{q} = 5.10 \Rightarrow \frac{3}{q} = 0.10 \Rightarrow 3 = 0.1q$

$q =$ 30 things

iii. For this scenario, you should be able to tell that profit increases at a constant rate (there is no maximum profit). How much additional profit does the company make when it sells each additional item?

$MR(q) = 7$, $MC(q) = 5 \Rightarrow MP(q) = 2$

'marginal profit' $=$ 2 dollars

(b) **The question below is unrelated to previous parts above.**
For a different company, the selling price is given by $p = 40 - 5x$ dollars/item for an order of x **hundred** items. In addition, you know the **total cost** is a **linear** function. The fixed cost is $FC = 10$ hundred dollars and it costs 22 hundred dollars to produce 3 hundred items. Thus, $TC(0) = 10$ and $TC(3) = 22$.

Find the quantity and selling price that maximize profit.
(Hint: First find the functions for $TR(x)$, $TC(x)$ and profit).

$TR(x) = 40x - 5x^2$

$TC(x) = m(x - x_0) + y_0 \quad (0,10), (3,22) \Rightarrow m = \frac{22-10}{3-0} = \frac{12}{3} = 4$

$TC(x) = 4x + 10$

$\text{PROFIT} = TR(x) - TC(x) = [40x - 5x^2] - [4x + 10]$

$\text{PROFIT} = -5x^2 + 36x - 10$

$x = -\frac{36}{2(-5)} = 3.6$

$p = 40 - 5 \cdot (3.6) = 22$

$x =$ 3.60 hundred items

$p =$ 22 dollars/item

5. (13 pts) The two parts below are unrelated.

(a) Grover invests \$3,000 in a bank account that pays simple interest. After 5 years, the account has earned \$1,215 in total interest. What is the annual interest rate on the account?

$F = P(1+rt) \Rightarrow 4215 = 3000(1+r\cdot 5)$

$1.405 = 1 + 5r$

$0.405 = 5r$

$r = \frac{0.405}{5} = 0.081$

8.1 %

(b) Shade the feasible region corresponding to the constraints:

$$2x + y \leq 70 \ , \quad 2x + 3y \leq 120 \ , \quad y \geq 12 \ , \quad x \geq 0.$$

Clearly, shade the feasible region and label ALL corners of the shaded region for full credit. (Show your work and your solving! Do NOT estimate from the picture, you must show the necessary algebra to solve for the appropriate intersections to get full credit).

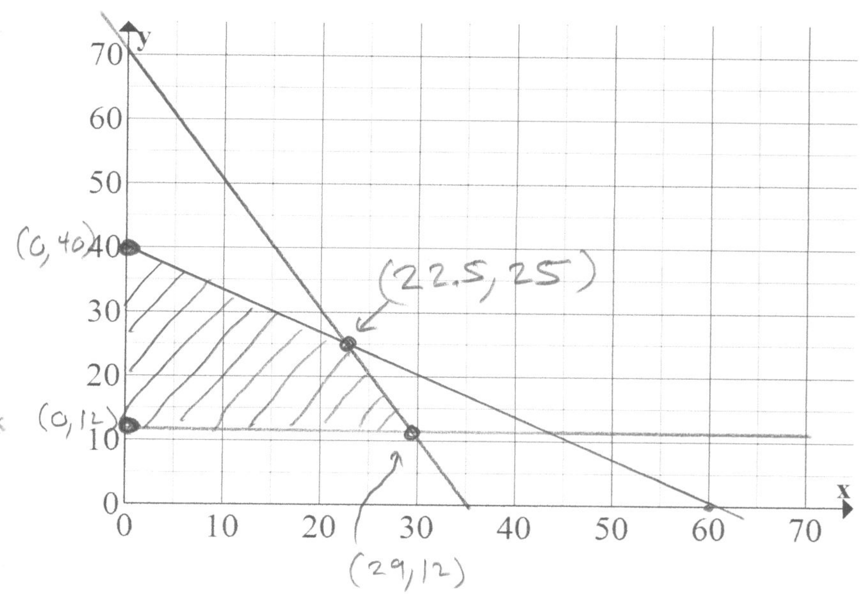
$2x + y \leq 70$ (35, 0) (0, 70)

$2x + 3y \leq 120$ (60, 0) (0, 40)

$\left.\begin{array}{l} y = 12 \\ 2x + y = 70 \end{array}\right\}$ $2x + 12 = 70$

$\Rightarrow 2x = 58$

$\Rightarrow x = 29$



$2x + y = 70 \Rightarrow y = 70 - 2x$

$2x + 3y = 120$

$2x + 3(70 - 2x) = 120$

$2x + 210 - 6x = 120$

$-4x = -90$

$x = 22.5$

$y = 70 - 2(22.5) = 25$

Corners of feasible region (list all): (0, 12), (0, 40), (29, 12), (22.5, 25)

6. (15 pts) (For all your work below, round your **final answer** to two digits after the decimal)

(a) Ann found an investment that will pay her 5% annual interest, compounded quarterly. How much must Ann invest in the account now so that she will have \$10,000 in five years?

$$F = P\left(1+\frac{r}{m}\right)^{mt} \Rightarrow 10000 = P\left(1+\frac{0.05}{4}\right)^{4\cdot 5}$$

$$10000 = P \cdot 1.0125^{20}$$

$$10000 = P \cdot 1.2820372$$

$$P = \frac{10000}{1.2820372} \approx 7800.0855$$

$7800.^{\underline{09}}$ dollars

(b) Molly deposits \$600 into an account that pays 4% annually, compounded continuously. How long will it take for the account balance to triple?

$$F = Pe^{rt} \Rightarrow 1800 = 600\, e^{0.04t}$$

$$3 = e^{0.04t}$$

$$\ln(3) = 0.04t$$

$$t = \frac{\ln(3)}{0.04} \approx 27.465307$$

27.47 years

(c) Sally buys a home for \$320,000. Six years later, she sells the home for \$400,000. What interest rate, compounded annually, did this investment represent for Sally?

$$F = P(1+r)^t \Rightarrow 400000 = 320000\,(1+r)^6$$

$$1.25 = (1+r)^6$$

$$(1.25)^{1/6} = 1+r$$

$$1.037890816 = 1+r$$

$$r = 0.037890816$$

3.79 %

7. (15 pts) (Round your **final answers** to two digits after the decimal)

(a) Samantha graduates with \$30,000 in student loans. Her loans have a 4% interest rate, compounded monthly. She will make her first payment at the end of this month and each month afterward for the next 10 years to pay off the entire loan. How big is each payment?

$$P = R\,\frac{1-(1+i)^{-n}}{i} \Rightarrow 30000 = R\,\underbrace{\frac{1-(1.00\overline{3})^{-120}}{0.00\overline{3}}}$$

$i = \frac{0.04}{12} = 0.00\overline{3}$

$30000 = R \cdot 98.770175$

$n = 12 \cdot 10 = 120$

$R = 303.73541$

$303.\underline{74}$ ______ dollars

(b) Oscar starts to save for retirement and he plans to retire in 30 years. At the end of each month he deposits \$500 in an account that earns 6% annually, compounded monthly. How much money will be in the account when Oscar retires and how much total interest did the account earn?

$$F = R\,\frac{(1+i)^n - 1}{i} \Rightarrow F = 500 \cdot \frac{(1.005)^{360}-1}{0.005}$$

$i = \frac{0.06}{12} = 0.005$ $\quad = 500 \cdot 1004.515042$

$= 502257.52$

$n = 30 \cdot 12 = 360$

INTEREST EARN $= F - \overbrace{R \cdot n}^{\text{TOTAL OF DEPOSITS}}$ ($500 \cdot 360$)

$= 502257.52 - 180{,}000 = 322{,}257.52$

Balance in 30 years = $502{,}257.\underline{52}$ ______ dollars

Total Interest Earned = $322{,}257.\underline{52}$ ______ dollars

(c) Today, Julie has \$10,000 in an investment earning 6% annually, compounded semiannually. In addition, at the beginning of every six month period, she makes deposits of \$400 into the same type of account. How much money does she have altogether at the end of 5 years?

$\boxed{\text{I}}$ $F = P\left(1+\frac{r}{n}\right)^{nt} \Rightarrow F = 10000\left(1+\frac{0.06}{2}\right)^{2\cdot 5} = 13{,}439.164$

$\boxed{\text{II}}$ $F = R\,\frac{(1+i)^n-1}{i}(1+i) = 400\,\frac{(1.03)^{10}-1}{0.03}(1.03) = 4{,}723.1183$

$i = \frac{0.06}{2} = 0.03$

TOTAL $= 18{,}162.282$

$n = 2 \cdot 5 = 10$

$18{,}162.\underline{28}$ ______ dollars