1. (13 points) A company sells items. The **total costs** are given by $TC(x) = \frac{1}{4}x^2 + 13x + 50$ dollars, and the **selling price**, p, for an order of x items is given by $p = 51 - \frac{3}{4}x$ dollars per item, where x is in items. Round your final answers to the nearest item or nearest cent.

(a) Find and simplify the formulas for total revenue, average cost and average variable cost.

$TR(x) = px$

$AC(x) = \frac{TC(x)}{x}$

$AVC(x) = \frac{VC(x)}{x}$

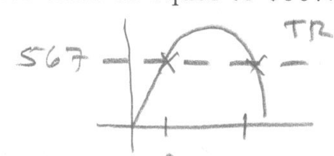
$TR(x) =$ $51x - \frac{3}{4}x^2$ dollars

$AC(x) =$ $\frac{1}{4}x + 13 + \frac{50}{x}$ dollars per item

$AVC(x) =$ $\frac{1}{4}x + 13$ dollars per item

(b) Find the range of quantities over which **total revenue** is greater than or equal to \$567.

$51x - \frac{3}{4}x^2 \stackrel{?}{=} 567$



$0 = \frac{3}{4}x^2 - 51x + 567$

$x = \frac{51 \pm \sqrt{51^2 - 4(\frac{3}{4})(567)}}{2(3/4)} = \frac{51 \pm \sqrt{900}}{(3/2)}$

$x = \frac{51 \pm 30}{(3/2)}$

$x = \frac{51 - 30}{(3/2)} = 14$

$x = \frac{51 + 30}{(3/2)} = 54$

from $x =$ 14 to $x =$ 54 items

(c) Find the quantity and selling price that give the maximum **profit**.

$\text{PROFIT} = TR(x) - TC(x)$

$= (51x - \frac{3}{4}x^2) - (\frac{1}{4}x^2 + 13x + 50)$

$= 51x - 3/4x^2 - 1/4x^2 - 13x - 50$

$= -x^2 + 38x - 50$

$x = -\frac{38}{2(-1)} = 19$

$p = 51 - \frac{3}{4}(19) = 36.75$

ASIDE: MAX PROFIT $= -(19)^2 + 38(19) - 50$
$= \$311$

$x =$ 19 items

$p =$ 36.75 dollars/item

2. (16 points) A company sells items. Total revenue is given by $TR(x) = 30x - 2x^2$ **hundred** dollars, the fixed costs are \$2300 (23 **hundred** dollars) and average variable costs are given by $AVC(x) = 3x^2 - 11x + 30$ dollars per item, where x is in **hundreds** of items. Round your **final answers** to the nearest item or nearest cent.

(a) What is the total cost to produce 200 **items**? (include units)

$TC(x) = 3x^3 - 11x^2 + 30x + 23$

$TC(2) = 3(2)^3 - 11(2)^2 + 30(2) + 23$

6300 dollars

63 Units: hundred dollars

EITHER ONE

(b) Find the longest interval over which $AVC(x)$ and $TR(x)$ are both increasing.

$AVC(x) = 3x^2 - 11x + 30$

INCREASING AFTER THE VERTEX

$x = -\frac{-11}{2(3)} = 1.8\overline{3}$

$TR(x) = 30x - 2x^2$

INCREASING BEFORE THE VERTEX

$x = -\frac{30}{2(-2)} = 7.5$

from $x =$ 1.83 to $x =$ 7.50

(c) Recall: $MR(x) = \dfrac{TR(x+0.01) - TR(x)}{0.01}$.
Find and *completely simplify* the formula for marginal revenue.

$$\frac{[30(x+0.01) - 2(x+0.01)^2] - [30x - 2x^2]}{0.01}$$

$$= \frac{30x + 0.3 - 2(x^2 + 0.02x + 0.0001) - 30x + 2x^2}{0.01}$$

$$= \frac{\cancel{30x} + 0.3 - \cancel{2x^2} - 0.04x - 0.0002 - \cancel{30x} + \cancel{2x^2}}{0.01}$$

$$= \frac{-0.04x + 0.2998}{0.01}$$

$MR(x) =$ $-4x + 29.98$

(d) Find a positive quantity at which total revenue equals variable cost.

$30x - 2x^2 \stackrel{?}{=} 3x^3 - 11x^2 + 30x$

$0 = 3x^3 - 9x^2 \;) \div x^2$

$0 = 3x - 9$

$9 = 3x$

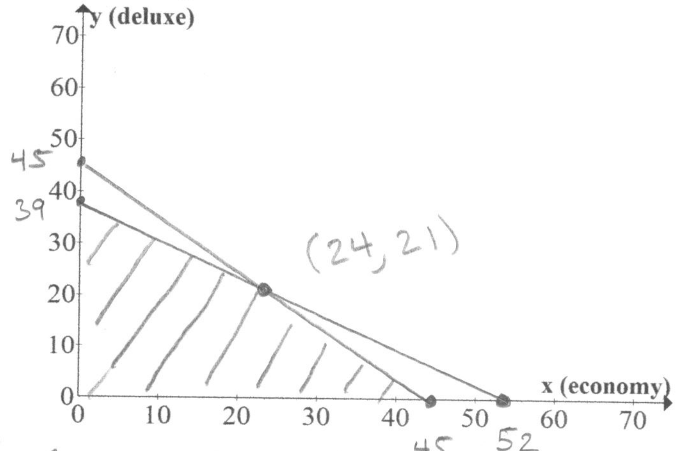
$x = 3$

$x =$ 3.00

3. (10 pts) The Wellbuilt Company manufactures two types of wood chippers, economy and deluxe. Each economy chipper requires 3 hours to assemble, 2 hours to paint, and results in \$30 of profit. Each deluxe chipper requires 4 hours to assemble, 2 hours to paint, and results in \$25 of profit. Today, the company is limit to at most 156 hours of assembly and at most 90 hours of painting. Let x be the number of **economy** wood chippers the company produces today, and let y be the number of **deluxe** wood chippers the company produces today.

(a) Give the constraints, then sketch and shade the feasible region (**clearly** label corners). You **must** show your algebra in finding all intersection points.

CHIPPERS =	x	y
ASSEMBLE =	3 hrs/chipper	4 hrs/chipper
PAINT =	2 hrs/chipper	2 hrs/chipper
PROFIT =	\$30/chipper	\$25/chipper



CONSTRAINTS: $3x + 4y \leq 156$ } (0, 39) (52, 0)
$2x + 2y \leq 90$ } (0, 45) (45, 0)

INTERSECTION: $2x + 2y = 90 \Rightarrow 2y = 90 - 2x \Rightarrow y = 45 - x$
$3x + 4y = 156 \Rightarrow 3x + 4(45 - x) = 156$
$3x + 180 - 4x = 156$
$-x = -24$
$x = 24$
$y = 21$

(b) How many units of each model should be produced to maximize profit? (Show the calculations to prove you checked all appropriate points)

PROFIT $= 30x + 25y$
$(0,0) \rightarrow 30(0) + 25(0) = \$0$
$(45,0) \rightarrow 30(45) + 25(0) = \$1350$
$(0,39) \rightarrow 30(0) + 25(39) = \$975$
$(24,21) \rightarrow 30(24) + 25(21) = \$1245$

$x =$ 45 economy wood chippers

$y =$ 0 deluxe wood chippers

4. (11 pts)

(a) For a certain commodity, the supply curve is **linear**.
The quantity supplied is $q = 10$ items, when the price is $p = \$10$/item, and the quantity supplied is $q = 40$ items, when the price is $p = \$100$/item.

i. Find and simplify the equation of the line for the supply curve.

$$m = \frac{100-10}{40-10} = \frac{90}{30} = 3$$

$$p = 3(q-10)+10 = 3q - 30 + 10$$

$p =$ $3q - 20$

ii. In addition, the demand curve is given by $pq + 20q = 1452$. Find the quantity and price that correspond to market equilibrium.

$$(3q-20)q + 20q = 1452$$

$$3q^2 - \cancel{20q} + \cancel{20q} = 1452$$

$$3q^2 = 1452$$

$$q^2 = 484$$

$$q = \pm 22$$

$$p = 3(22) - 20 = 46$$

$q =$ 22 items

$p =$ 46 dollars/item

(b) Solve $8(1 + e^{2t}) - 5 = 19$.
Give your final answer **as a decimal**, accurate to three digits after the decimal.

$$8(1+e^{2t}) = 24$$

$$1 + e^{2t} = 3$$

$$e^{2t} = 2$$

$$2t = \ln(2)$$

$$t = \frac{\ln(2)}{2} \approx 0.346574$$

$t =$ 0.347