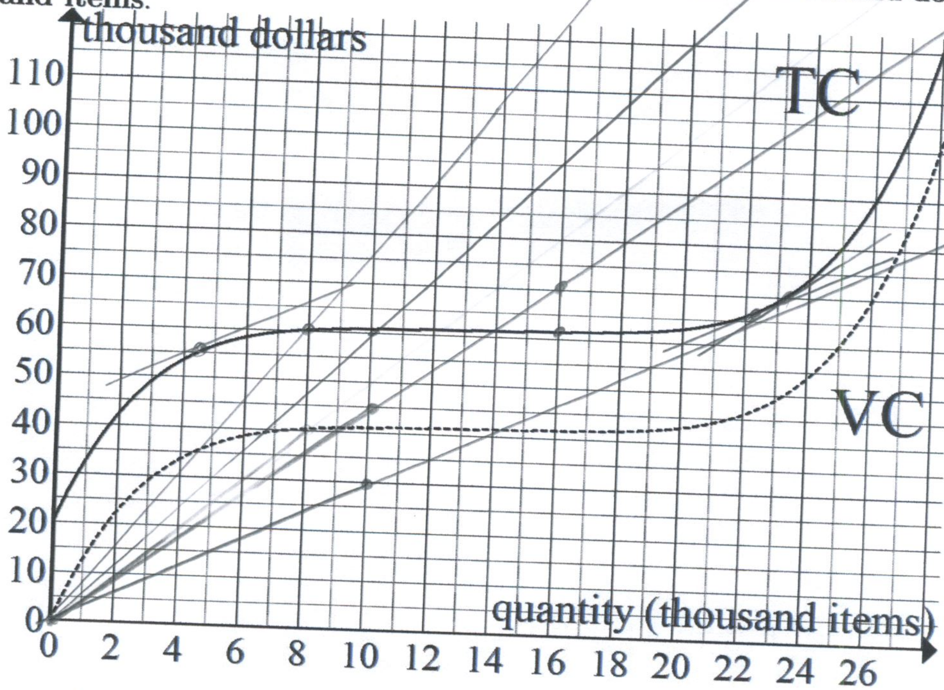
PART (b) PART (a) SLOPE = 6

PART (d) TR

1. (16 points) The following graph shows total cost and variable cost in **thousand dollars** for selling x **thousand items**.



PART (c) SLOPE = 3

For each part, clearly explain and label your work. Be as accurate as possible.

(a) Find the quantity at which average variable cost is equal to 6 dollars/item.

DRAW REFERENCE LINE WITH SLOPE 6
(0,0) (10, 60)

AVC(x) = 6 AT ABOUT x=6.6

$x =$ 6.6 thousand items

(b) Find the average cost to produce $x = 8$ thousand items (give units for your answer).

$AC(8) = \frac{TC(8)}{8} \approx \frac{60}{8} = 7.5$

$AC(8) =$ 7.5 dollars/item

(c) Find all quantities at which **marginal cost** is 3 dollars per item.

DRAW REFERENCE LINE WITH SLOPE 3
(0,0) (10,30)

~4.5 , 22

(list answers) $x =$ 4.5, 22 thousand items

(d) For both parts below, the selling price is fixed at $p = 4.50$ dollars/item. → (10, 45)

i. At what quantity is profit maximized?

MATCH SLOPES

$x =$ 23 thousand items

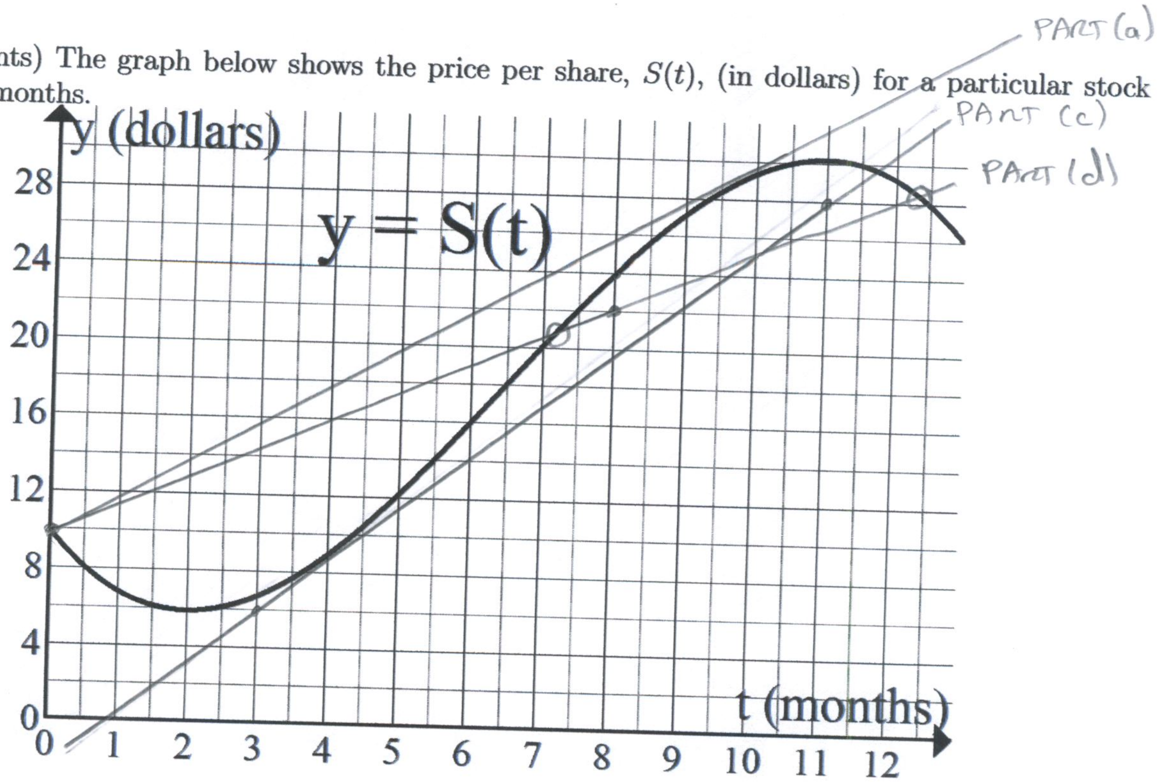
ii. What is the profit if you sell 16 thousand items?

TR(16) − TC(16)

≈ 71 − 62 = 9

Profit = 9 thousand dollars

2. (14 points) The graph below shows the price per share, $S(t)$, (in dollars) for a particular stock after t months.



For each part, clearly label your work in the graph and show your computations in the problems.

(a) Find the largest overall rate of change (That is, find the largest value of $\frac{S(t)-S(0)}{t}$).

TWO POINTS : (0, 10) (5, 19.5)

$\text{SLOPE} \approx \frac{19.5-10}{5-0} = \frac{9.5}{5}$

1.90 dollars per month

(b) Compute the change in stock price over the first 8 months.

$S(8) - S(0) = 24 - 10 = 14$

14 dollars

(c) As accurately as possible, estimate the rate of change in the stock value over the first day of the 4th month. In other words, what is the rate from 4 to about 4.03? (Give units)

TWO POINTS : (3, 6) (11, 28)

$\text{SLOPE} \approx \frac{28-6}{11-3} = \frac{22}{8} = 2.75$

$\frac{S(4.03)-S(4)}{0.03} \approx$ 2.75 dollars per month

(d) Another stock, $R(t)$, also starts at a value of 10 dollars, but it increases at a constant rate of 1.50 dollars per month. Find all times (after $t = 0$) when the two stocks have the same value.

$8 \cdot 1.50 = 12$, $10 + 12 = 22$

GOES THRU (0, 10) AND (8, 22)

(list answers) $t =$ 7.2, 12.3 months

3. (13 pts) You sell Things.
The price per Thing, p, on an order of q Things is given by $p = 35 - 0.25q$.
The total cost is a **linear** function that has a fixed cost of 100 and you also know that if you produce $q = 2$ Things, then the total cost is $TC(2) = 135$ dollars.

(a) Find the formulas for Total Revenue and Total Cost.

"slope of TC" $= \frac{135-100}{2-0} = 17.5$

$TR(q) =$ $35q - 0.25q^2$ dollars

$TC(q) =$ $17.5q + 100$ dollars

(b) Find and completely simplify the formulas for Marginal Revenue and Marginal Cost.

$MR(q) = [35(q+1) - 0.25(q+1)^2] - [35q - 0.25q^2]$

$= \cancel{35q} + 35 - 0.25(q^2+2q+1) - \cancel{35q} + 0.25q^2$

$= 35 - \cancel{0.25q^2} - 0.5q - 0.25 + \cancel{0.25q^2}$

$= 34.75 - 0.5q$

$MR(q) =$ $-0.5q + 34.75$ dollars per Thing

$MC(q) =$ 17.5 dollars per Thing

(c) Find all quantities at which total revenue is 15 dollars more than total cost.
(Round your answers to the nearest Things).

$35q - 0.25q^2 \stackrel{?}{=} 17.5q + 100 + 15$

$-0.25q^2 + 17.5q - 115 \stackrel{?}{=} 0$

$q = \frac{-17.5 \pm \sqrt{17.5^2 - 4(-0.25)(-115)}}{2(-0.25)} = \frac{-17.5 \pm \sqrt{306.25 - 115}}{-0.5}$

$q = \frac{-17.5 \pm \sqrt{191.25}}{-0.5} = \frac{-17.5 \pm 13.82931669}{-0.5} = \begin{cases} 7.341366628 \\ 62.65863338 \end{cases}$

(list all): $q =$ 7, 63 Things

4. (11 pts) You sell shirts. For this problem, x is in hundreds of shirts.
The total revenue is $TR(x) = 23x - x^2$ hundred dollars.
The total cost is $TC(x) = x^2 + 8x + 12$ hundred dollars.
(Round all final answers to the nearest dollar or nearest shirt, appropriately).

(a) Find the fixed cost and give the formulas for Average Cost and price per item.

$FC =$ 12 hundred dollars

$AC(q) =$ $x + 8 + \frac{12}{x}$ dollars per shirt

$p =$ $23 - x$ dollars per shirt

(b) Find all quantities at which average cost is equal to 15 dollars per shirt.

$$x + 8 + \frac{12}{x} = 15$$

$$x^2 + 8x + 12 = 15x$$

$$x^2 - 7x + 12 = 0 \rightarrow \text{OR USE QUADRATIC FORMULA}$$

$$(x-3)(x-4) = 0$$

(list all) $x =$ 3, 4 hundred shirts

(c) What quantity and price maximize profit?

$$\text{PROFIT} = (23x - x^2) - (x^2 + 8x + 12)$$

$$\text{PROFIT} = -2x^2 + 15x - 12$$

$$x = -\frac{15}{2(-2)} = 3.75$$

$$p = 23 - 3.75 = 19.25$$

quantity = 3.75 hundred shirts

price = 19.25 dollars per shirt

5. (10 points) Your company manufactures two types of chairs: metal chairs and plastic chairs. Each metal chair requires 3 hours of labor to assemble and 2 hours of labor to paint. Each plastic chair requires 1/2 hour of labor to assemble and 1 hour of labor to paint. The profit for each metal chair is \$15 and the profit for each plastic chair is \$12.

The maximum number of hours of labor available to assemble the chairs is 150 hours each day. The maximum number of hours of labor available to paint the chairs is 168 hours each day.

Let x be the number of metal chairs you produce in a day and y be the number of plastic chairs you produce in a day.

(a) Give the constraints, then sketch and shade the feasible region. (Show your work for how you found ALL the necessary points).

TOTAL ASSEMBLY HOURS $= 3x + \frac{1}{2}y \leq 150$

TOTAL PAINT HOURS $= 2x + y \leq 168$

INTERSECTION:

(ii) $2x + y = 168 \Rightarrow y = 168 - 2x$

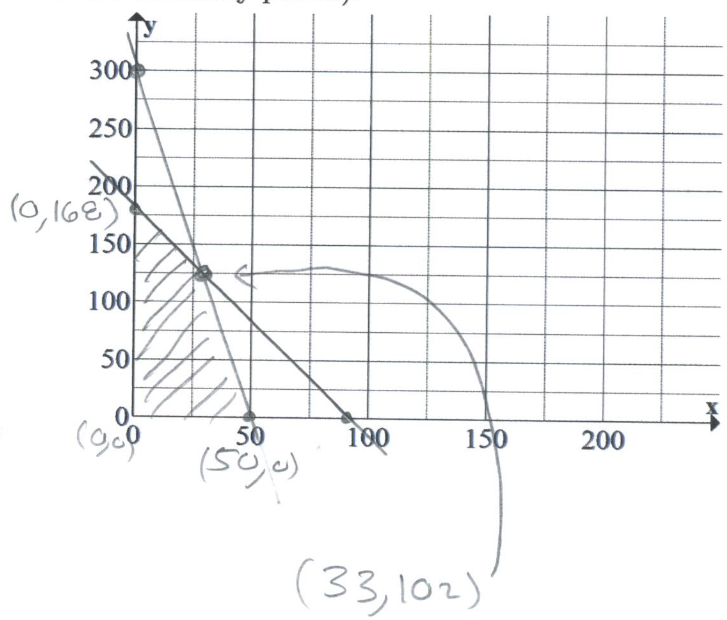
(ii) INTO (i) $\Rightarrow 3x + \frac{1}{2}(168 - 2x) \stackrel{?}{=} 150$

$3x + 84 - x = 150$

$2x = 66$

$x = 33$

$y = 168 - 2(33) = 102$



(b) How much of each type of chair should you produce to give maximum profit? Also give the value of maximum profit? (Show your work)

PROFIT $= 15x + 12y$

$(0,0) \Rightarrow$ PROFIT $= 15(0) + 12(0) = 0$

$(50,0) \Rightarrow$ PROFIT $= 15(50) + 12(0) = \$750$

$(0,168) \Rightarrow$ PROFIT $= 15(0) + 12(168) = \$2016$

$(33,102) \Rightarrow$ PROFIT $= 15(33) + 12(102) = \$1719$

$x =$ 0 metal chairs

$y =$ 168 plastic chairs

Max Profit $=$ 2,016 dollars

6. (12 points)

(a) Suppliers are willing to produce 96 items if the price is \$410/item and 136 items if the price is \$540/item. The supply curve is **linear**.

i. Give the equation of the line for the supply curve. (Use p for price and q for quantity).

$$\text{SLOPE} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{540-410}{136-96} = \frac{130}{40} = 3.25$$

$$\left.\begin{aligned} p &= 3.25(q-96)+410 \\ p &= 3.25q+98 \end{aligned}\right\}$$

$p =$ $3.25q + 98$

ii. You are also told that the demand curve is $2p + 6q = 921$.
Find the quantity and price that corresponds to market equilibrium.

$$2(3.25q+98)+6q = 921$$
$$6.5q + 196 + 6q = 921$$
$$12.5q = 725$$
$$q = 58$$
$$p = 3.25(58)+98 = 286.5$$

$q =$ 58 items

$p =$ 286.50 dollars/item

(b) Which account is better?
Account A: 3.97% annually, compounded continuously, or
Account B: 4% annually, compounded quarterly
Explain your answer (by computing appropriate numbers or explaining in some other way):

$\boxed{A}$ $APY = \left[e^{0.0397} - 1\right] \times 100\% = 4.049857779\%$

$\boxed{B}$ $APY = \left[\left(1+\frac{0.04}{4}\right)^4 - 1\right] \times 100\% = 4.060401\%$ ← BETTER

Circle the one that is better: Account A or Account B

7. (12 points) (For all your work below, round your **final answer** to two digits after the decimal)

(a) Roger deposits \$1000 into an account that pays 5% annually, compounded continuously. How long will it take for him to earn \$600 in **interest**?

$F = P + I = 1000 + 600 = 1600$

$1600 = 1000\, e^{0.05t}$

$1.6 = e^{0.05t}$

$\ln(1.6) = 0.05t$

$t = \frac{\ln(1.6)}{0.05} \approx 9.400072585$

$t =$ 9.40 years

(b) Steffi invests \$5000 into an account where the interest is compounded semi-annually. The balance of the account double in 9 years. What was her semi-annual interest rate?

$\underbrace{10{,}000}_{\text{double}} = 5000\left(1+\frac{r}{2}\right)^{2\cdot 9}$

$2 = \left(1+\frac{r}{2}\right)^{18}$

$2^{1/18} = 1 + \frac{r}{2}$

$1.039259226 = 1 + \frac{r}{2}$

$0.039259226 = r/2$

$\Rightarrow r = 0.07851845 2$

7.85 %

(c) Andre invests \$15,000 in a CD certificate that pays 8% annual simple interest for 5 years. When the CD matures (at the end of the 5 years), he takes all the money and invests it into an new account that pays 7% annually, compounded quarterly for an additional 5 years. What is the ending balance of the new account?

1st 5 years : $F = 15000(1 + 0.08 \cdot 5) = 21000$

2nd 5 years : $F = 21000\left(1 + \frac{0.07}{4}\right)^{4.5}$

$= 21000(1.0175)^{20}$

$= 29710.34211$

29,710.34 dollars

8. (12 points) (Round your **final answers** to the nearest dollar or year)

(a) Serena starts saving for retirement. She plans to invest in a retirement account that earns 6% annually, compounded monthly. Today, she plans to start making equal monthly payments at the beginning of each month and she wants to have a balance of \$3,000,000 in her account in 35 years. Find the size of the monthly payments.

DUE, FV, FIND R, $i = \frac{0.06}{12} = 0.005$, $n = 12 \cdot 35 = 420$ payments.

$$3{,}000{,}000 = R \, \frac{(1.005)^{420} - 1}{0.005} (1.005)$$

$$3{,}000{,}000 = R \cdot 1431.83385$$

$$R = 2095.215167$$

2095 dollars

(b) Rafa just finished paying off his home loan. The loan balance was earning 4% annual interest, compounded monthly. Rafa made payments of \$2,000 at the end of each month for 20 years. What was the starting balance of the loan? And how much total interest did Rafa pay?

LOAN ⇒ ORDINARY, PV, $i = \frac{0.04}{12} = 0.00\overline{3}$, $n = 12 \cdot 20 = 240$ payments

$$P = 2000 \, \frac{1 - (1.00\overline{3})^{-240}}{0.00\overline{3}} = 2000 \cdot 165.021864 = 330043.728$$

$$\text{TOTAL PAID} = R \cdot n = 2000 \cdot 240 = 480{,}000$$

$$\text{TOTAL INTEREST PAID} = 480000 - 330044 = 149956$$

Starting Balance = 330,044 dollars

Total Interest Paid = 149,956 dollars

(c) Pete has \$3,600,000 saved in his retirement account when he retires early at age 30. The money is in an account that earns 9% annually, compounded quarterly. He plans to withdraw \$90,000 from the account at the end of each quarter.
How old will Pete be when the money is all gone?

ORDINARY, PV, $i = \frac{0.09}{4} = 0.0225$, $n = 4t$

$$3{,}600{,}000 = 90{,}000 \, \frac{1 - (1.0225)^{-4t}}{0.0225}$$

$$40 = \frac{1 - (1.0225)^{-4t}}{0.0225}$$

$$0.9 = 1 - (1.0225)^{-4t}$$

$$(1.0225)^{-4t} = 0.1$$

$$-4t \ln(1.0225) = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-4\ln(1.0225)} \approx 25.87103458 \approx 26 \text{ years from now}$$

$30 + 26 = 56$

Pete will be 56 years old