1. (16 points) Suppose a company has fixed costs of \$43,000 (430 hundred dollars) and average variable costs given by $AVC(x) = x^2 - 5x + 10$ dollars per item, where x is in hundreds of items. Suppose further that the selling price of its product is $p = 190 - \frac{4}{3}x$ dollars per item.

Round your **final answers** to the nearest item or nearest cent.

(a) (4 pts) Find and simplify the formulas for variable cost, total cost, average cost, and total revenue.

$VC(x) =$ $x^3 - 5x^2 + 10x$ hundred dollars

$TC(x) =$ $x^3 - 5x^2 + 10x + 430$ hundred dollars

$AC(x) =$ $x^2 - 5x + 10 + \frac{430}{x}$ dollars per item

$TR(x) =$ $190x - \frac{4}{3}x^2$ hundred dollars

(b) (4 pts) Find the shutdown price.

"x-coord. of vertex of AVC" $= -\frac{(-5)}{2(1)} = 2.50$ hundred items

"y-coord of vertex of AVC" $= (2.5)^2 - 5(2.5) + 10$

$= 3.75$ dollars/item

$SDP =$ 3.75 dollars/item

(c) (4 pts) Find the **selling price** that gives the maximum total revenue.

"x-coord. of max total revenue" $= -\frac{190}{2(-4/3)} = 71.25$ hundred items

"corresponding selling price" $= p = 190 - \frac{4}{3}(71.25) = 95$ dollars/item

$p =$ 95 dollars/item

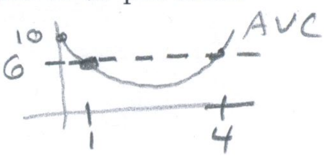
(d) (4 pts) Find the range of quantities over which $AVC(x)$ is less than or equal to \$6 per item.

$x^2 - 5x + 10 = 6$

$x^2 - 5x + 4 = 0$ → OR USE QUAD. FORMULA

$(x-1)(x-4) = 0$

$x = 1, 4$



from $x =$ 1 to $x =$ 4 hundred items

2. (13 points) Consider the two functions

$$f(x) = \frac{3}{4}x^2 - 3x + 2 \quad \text{and} \quad g(x) = x^2 - 8x + 15.$$

(a) (5 pts) Find and completely simplify $\frac{g(x+h)-g(x)}{h}$.

$$= \frac{[(x+h)^2 - 8(x+h) + 15] - [x^2 - 8x + 15]}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{8x} - 8h + \cancel{15} - \cancel{x^2} + \cancel{8x} - \cancel{15}}{h}$$

$$= \frac{2xh + h^2 - 8h}{h}$$

$$= 2x + h - 8$$

$\frac{g(x+h)-g(x)}{h} =$ $2x+h-8$

(b) (4 pts) Find all solutions to $g(x) - f(x) = 4$.

$$[x^2 - 8x + 15] - [\tfrac{3}{4}x^2 - 3x + 2] \overset{?}{=} 4$$

$$x^2 - 8x + 15 - \tfrac{3}{4}x^2 + 3x - 2 \overset{?}{=} 4$$

$$\tfrac{1}{4}x^2 - 5x + 13 = 4$$

$$\tfrac{1}{4}x^2 - 5x + 9 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 4(\frac{1}{4})(9)}}{2(\frac{1}{4})}$$

$$\rightarrow x = \frac{5 \pm 4}{\frac{1}{2}}$$

$$x = \frac{5-4}{(\frac{1}{2})} = 2$$

$$x = \frac{5+4}{\frac{1}{2}} = 18$$

(List solutions) $x =$ $2, 18$

(c) (4 pts) Find the longest interval over which $f(x)$ and $f(x) - g(x)$ are both increasing.

$f(x) = \frac{3}{4}x^2 - 3x + 2$

"x-coord. of vertex" $= \frac{-(-3)}{2(3/4)} = 2$

INCREASING AFTER $x = 2$

$f(x) - g(x) = -\frac{1}{4}x^2 + 5x - 13$

"x-coord. of vertex" $= \frac{-5}{2(-\frac{1}{4})} = 10$

INCREASING BEFORE $x = 10$

from $x =$ 2 to $x =$ 10

3. (10 points) On this page you will solve the following linear programming problem:
Maximize $f(x, y) = 4x + 5y$ subject to the constraints

$$\underset{①}{x + y \leq 7.5} \ , \quad \underset{②}{2.25x + y \leq 12} \ , \quad \underset{③}{y \leq 6} \ , \quad x \geq 0 \ , \quad y \geq 0.$$

(a) (8 pts) Accurately sketch the feasible region.
CLEARLY, shade the feasible region and label ALL corners of the feasible region for full credit.
(Show you work! Do NOT try to estimate the coordinates of the corners from the picture, you MUST show the necessary algebra to solve for the appropriate intersections to get full credit).

$x + y = 7.5 \Rightarrow (0, 7.5), (7.5, 0)$

$2.25x + y = 12 \Rightarrow (0, 12) \quad (5.\bar{3}, 0)$

INTERSECTION OF ① \& ③

$y = 6$

$x + y = 7.5$

$\Rightarrow x + 6 = 7.5 \Rightarrow x = 1.5$, $y = 6$

INTERSECTION OF ① \& ②

$x + y = 7.5 \Rightarrow y = \underbrace{7.5 - x}$

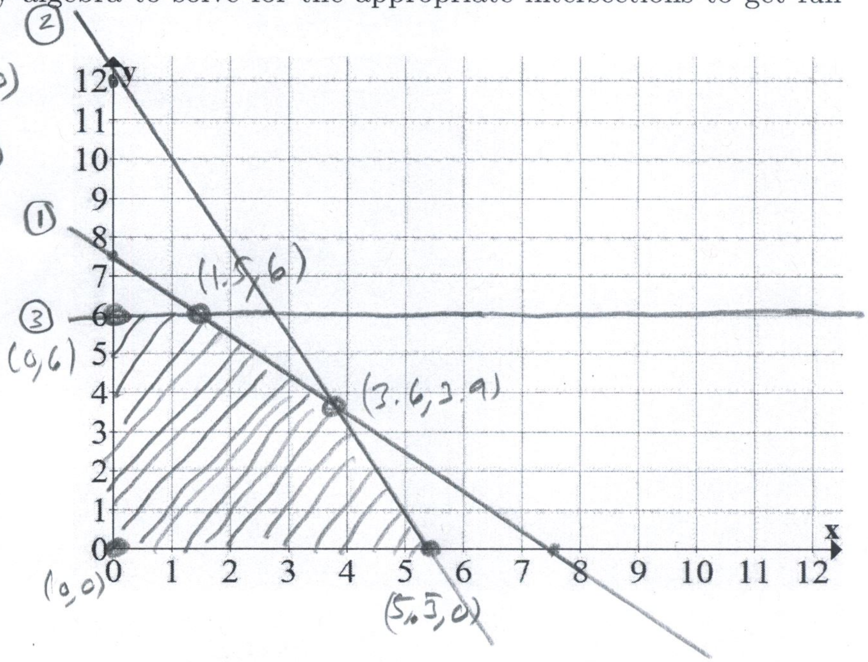
$2.25x + y = 12$

$2.25x + 7.5 - x = 12$

$1.25x = 4.5$

$x = \frac{4.5}{1.25} = 3.6$

$y = 7.5 - 3.6 = 3.9$



CORNERS: $(0,0)$

$(5.\bar{3}, 0)$ +4

$(0, 6)$

$(1.5, 6)$ +2

$(3.6, 3.9)$ +2

(b) (2 pts) What is the maximum value of $f(x, y) = 4x + 5y$ subject to the given constraints?

$f(0,0) = 4(0) + 5(0) = 0$

$f(5.\bar{3}, 0) = 4(5.\bar{3}) + 5(0) = 21.\bar{3}$

$f(0, 6) = 4(0) + 5(6) = 30$

$f(1.5, 6) = 4(1.5) + 5(6) = 36$

$f(3.6, 3.9) = 4(3.6) + 5(3.9) = 33.9$

Max $f(x, y)$ value = 36

4. (11 pts)

(a) The supply curve is $p - q = 10$ ① and the demand curve is $q(2p - 10) = 5500$ ②, where q is in items.

i. (4 pts) Give the quantity and price that corresponds to market equilibrium.

① $p = q + 10$

① INTO ② $q\,(2(q+10)-10) \overset{?}{=} 5500$

$q\,(2q+20-10) = 5500$

$2q^2 + 10q = 5500$

$2q^2 + 10q - 5500 = 0$

$q = \dfrac{-10 \pm \sqrt{100 - 4(2)(-5500)}}{2(2)} = \dfrac{-10 \pm \sqrt{44100}}{4}$

$q = \dfrac{-10 \pm 210}{4} = \begin{cases} \dfrac{-10-210}{4} = \cancel{-55} \\ \dfrac{-10+210}{4} = 50 \end{cases}$

$p = (50) + 10 = 60$

$(q, p) =$ (50, 60)

ii. (3 pts) If the market price is \$30 per item is there a shortage or surplus? And how many items is the shortage or surplus?

DEMAND : $p = 30 \Rightarrow q\,(2(30)-10) = 5500 \Rightarrow 50q = 5500 \Rightarrow q = 110$ item

SUPPLY : $p = 30 \Rightarrow 30 - q = 10 \Rightarrow q = 20$ item

$110 - 20 = 90$ item shortage

↑ DEMANDED ↑ SUPPLIED

Circle one: SHORTAGE or SURPLUS

By this many: 90 items

(b) (4 pts) Solve $21 - 3e^{0.1t} = 9$. (1.05)

Give your final answer as a decimal, accurate to three digits after the decimal.

$-3(1.05)^{0.1t} = -12$

$(1.05)^{0.1t} = 4$

$\ln(1.05^{0.1t}) = \ln(4)$

$0.1t \ln(1.05) = \ln(4)$

$t = \dfrac{\ln(4)}{0.1 \ln(1.95)} = \dfrac{1.38694361}{0.004879016} = 284.1339817$

$\boxed{t = 284.134}$