MATH 111
Final Exam
Autumn 2018

Name _____

Student ID #_____ Section _____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- Check that your exam contains **7** problems and put your name at the top of each page (**front and back**).
- You are allowed to use a TI-30XIIS calculator, a ruler, and a 8.5-inch-by-11-inch sheet containing hand-written notes. All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- **You must show your work.** Clearly label lines and points that you are using and show all calculations. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- When rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the back of the last page and **indicate that you have done so**. If you still need more room, ask your TA for an extra page to staple to your exam.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.

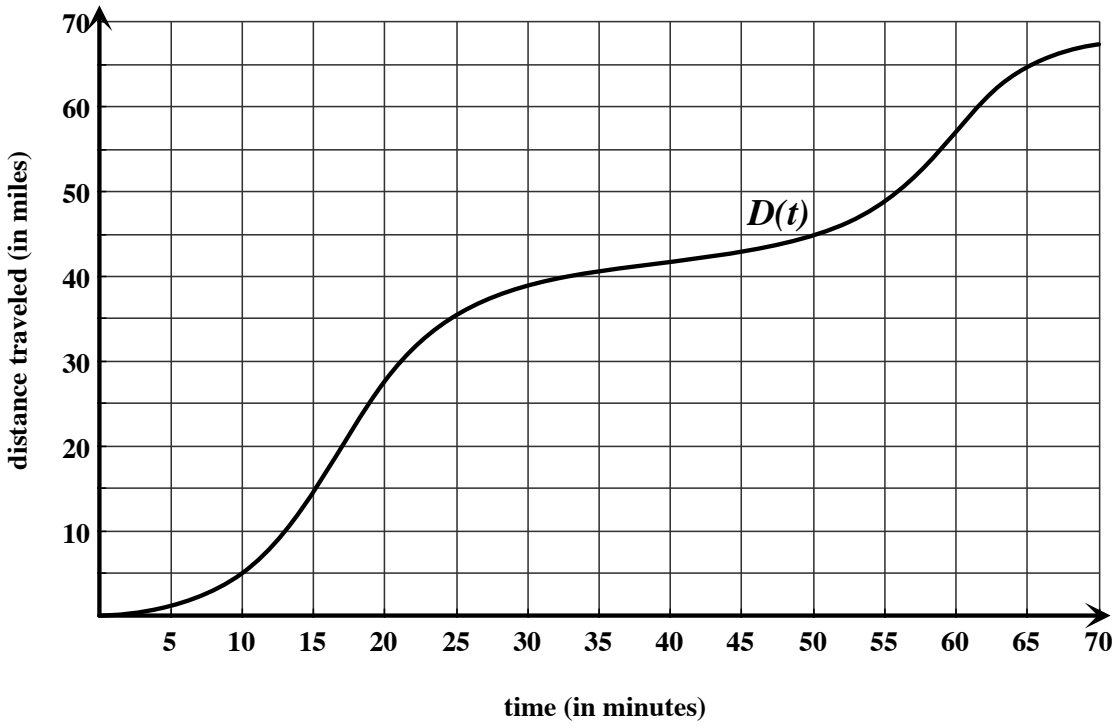
GOOD LUCK!

Suppose you produce and sell Things. The following table summarizes the terms we've learned so far relating to revenue and cost. Assume you are given a graph of total cost $TC(q)$ and total revenue $TR(q)$ for producing and selling q Things.

Term	**Definition**	**Related equations and formulas**	**Graphical Interpretation**
total cost $TC(q)$	the total amount you spend to produce q Things	$TC(q) = VC(q) + FC$	—
variable cost $VC(q)$	the money you spend to produce q Things without including fixed costs	$VC(q) = TC(q) - FC$	the graph of VC has the same shape as TC and goes through the origin
fixed cost FC	the money you must spend even if you produce 0 Things; also known as *overhead*	$FC = TC(q) - VC(q)$ $FC = TC(0)$	the vertical distance between the TC and VC graphs OR the "y"-intercept of the TC graph
average cost $AC(q)$	total cost averaged over the number of Things produced	$AC(q) = \frac{TC(q)}{q}$	the slope of the diagonal line through the TC graph at q
average variable cost $AVC(q)$	variable cost averaged over the number of Things produced	$AVC(q) = \frac{VC(q)}{q}$	the slope of the diagonal line through the VC graph at q
breakeven price BEP	the smallest value of average cost	—	the slope of the least steep diagonal line that intersects the TC graph
shutdown price SDP	the smallest value of average variable cost	—	the slope of the least steep diagonal line that intersects the VC graph
marginal cost $MC(q)$ (see footnote)	the incremental rate of change in TC from q to $q+1$ Things	$MC(q) = \frac{TC(q+1)-TC(q)}{1}$	the slope of the secant line through TC (or VC) at q and $q+1$
total revenue $TR(q)$	the total amount you receive when you sell q Things	—	—
average revenue $AR(q)$	total revenue averaged over the number of Things sold; also known as *price per Thing*	$AR(q) = \frac{TR(q)}{q}$	the slope of the diagonal line through the TR graph at q
marginal revenue $MR(q)$ (see footnote)	the incremental rate of change in TR from q to $q+1$ Things	$MR(q) = \frac{TR(q+1)-TR(q)}{1}$	the slope of the secant line through the TR graph at q and $q+1$
profit $P(q)$	the money you are left with after subtracting total cost from total revenue	$P(q) = TR(q) - TC(q)$	the vertical distance between TR and TC (when $TR > TC$)

NOTE: If q is measured in hundreds or thousands of Things, the definitions, formulas, and graphical interpretations of marginal revenue and marginal cost must be adjusted appropriately.

1. (15 points) The following is the graph of distance traveled vs. time for a car on a long straight road.



(a) What is the car's average speed during the forty-minute interval beginning at $t = 10$?

ANSWER: _____mpm

(b) What is the car's largest average trip speed?

ANSWER: _____mpm

(c) Find all values of h for which the following is true: The car travels 15 miles in the h-minute interval beginning at $t = 32.5$.

ANSWER: (list all) $h =$_____

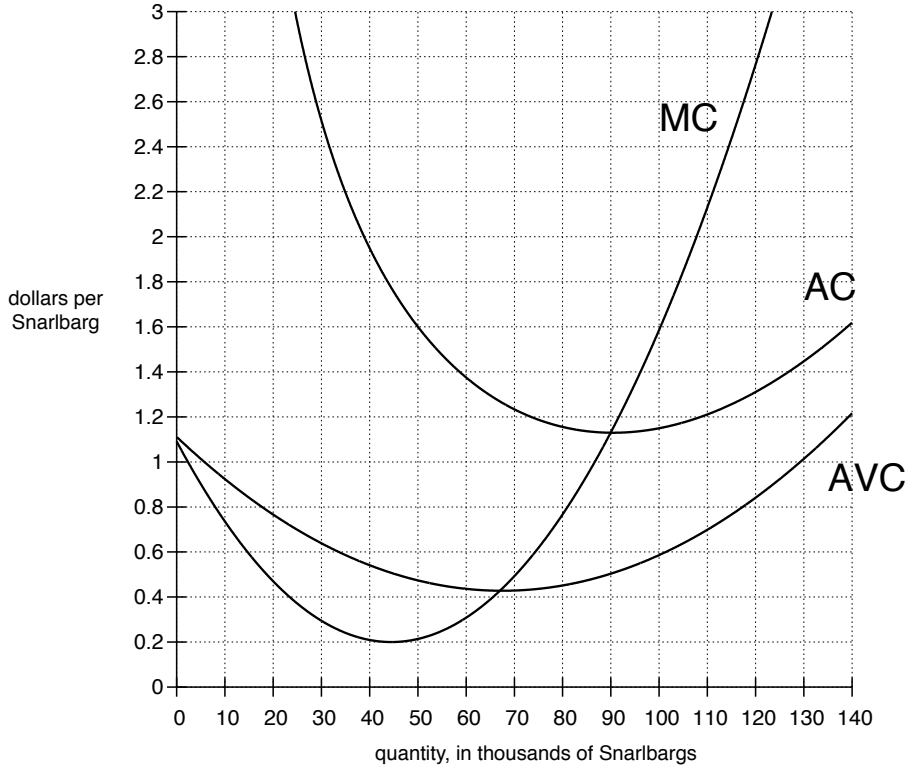
(d) Find all values of t for which $\dfrac{D(t)}{t} = 1.25$.

ANSWER: (list all) $t =$_____minutes

(e) Find all values of t such that $\dfrac{D(t+0.01) - D(t)}{0.01} = 1.25$.

ANSWER: (list all) $t =$_____minutes

2. (15 points) You make and sell Snarlbargs. Your average cost (AC), average variable cost (AVC) and marginal cost (MC) functions are graphed below.



(a) What is the lowest value of average variable cost?

ANSWER: _____dollars per Snarlbarg

(b) What is your breakeven price?

ANSWER: _____dollars per Snarlbarg

(c) How much does **total cost** increase if quantity changes from 20,000 to 20,001 Snarlbargs? Include units.

ANSWER: _____UNITS: _____

(d) If you sell Snarlbargs for \$0.80 each, what quantity maximizes your profit?

ANSWER: _____thousand Snarlbargs

(e) If instead you sell Snarlbargs for \$2.40 each, what is your maximum profit? Include units.

ANSWER: _____UNITS: _____

3. (15 points) You sell Kleems. Your **total revenue** and **total cost** in dollars for selling q Kleems are given by the formulas:

$$TR(q) = -0.12q^2 + 57.6q \text{ and } TC(q) = 16q + 1930.$$

(a) What is the longest interval on which **total revenue is increasing** and **profit is positive**? (Round your answers to the nearest whole Kleem.)

ANSWER: from $q =$_____ to $q =$_____ Kleems

(b) Use the definition $MR(q) = \dfrac{TR(q+1) - TR(q)}{1}$ to find the formula for **marginal revenue**.

ANSWER: $MR(q) =$ _____

(c) What quantity yields the largest possible **profit**? (Round your answer to the **nearest whole Kleem**.)

ANSWER: $q =$_____ Kleems

4. (12 points) In each of the following, your total revenue and total cost are **linear functions** of quantity q.

   (a) You make and sell hydraulic refrigerator lifts. You sell the lifts for \$435 each. Each lift costs \$270 to produce and your fixed cost is \$51,480. At what quantity will you break even?

   ANSWER: _____lifts

   (b) You make and sell dishwashers. Each dishwasher costs \$310 to produce. You sell each one for \$880. What would your fixed cost need to be to break even at a quantity of 100 dishwashers?

   ANSWER: _____dollars

   (c) You make and sell fancy fish tanks. Suppose your marginal cost is a constant m dollars per tank. Your fixed cost is \$76,500, you sell tanks for \$1280 per tank, and you break even at a quantity of 300 tanks. What is your marginal cost?

   ANSWER: _____dollars per tank

   (d) You make and sell crab gymnasiums. Each gym costs \$225 to produce and your fixed cost is \$82,000. At a quantity of 1000 gyms, total revenue **exceeds** total cost by \$8000. At what price do you sell each crab gym?

   ANSWER: _____dollars per gym

5. (15 points)

(a) In an account compounded continuously, what annual interest rate results in an APY of 5.8%?

ANSWER: _____%

(b) In an account earning 2.75% annual simple interest, how much must you deposit today to have \$8000 four years from now?

ANSWER: \$_____

(c) In an account paying 9% annually, compounded quarterly, how long does it take to double your principal?

ANSWER: _____years

(d) In an account earning 5.2% annually, compounded continuously, what is the percentage change in the balance over a 7-month period?

ANSWER: _____%

(e) How much **interest** does \$20,000 earn in 15 years in an account earning 1.32%, compounded three times per year?

ANSWER: \$_____

6. (16 points) Alice, Bernard, and Cecelia begin new jobs today. Each has a starting salary of \$65,000 per year. As long as they are employed at their respective companies, they will receive raises according to the following schedule:

   Alice: salary will increase by \$1500 every year

   Bernard: salary will increase by 2% every year

   Cecelia: salary will increase by 1% every year **except** every fifth year, when her salary will increase by 10%.

   As always, you must show all work. A correct answer with no justification will receive no credit.

   (a) Whose salary is highest during their fourth year of employment (i.e., after 3 raises)?

   ANSWER: _____

   (b) Whose salary is highest during their 11^{th} year of employment (i.e., after 10 raises)?

   ANSWER: _____

   (c) During which year of employment does Alice's salary exceed \$100,000?

   ANSWER: during their _____year

   (d) During which year of employment does Bernard's salary exceed \$100,000?

   ANSWER: during their _____year

7. (12 points) Gerry is saving to buy a house four years from now. After researching housing-market and interest rate trends, Gerry predicts that the house will cost approximately \$650,000 and will require a down-payment equal to 20% of the selling price of the house.

   (a) If Gerry makes equal payments at the end of each month for four years to a savings account paying 1.2% interest compounded monthly, what size should the payments be so that Gerry can pay a 20% down-payment on a house that costs \$650,000?

   ANSWER: \$_____

   (b) Gerry meets the goal and makes a 20% down-payment on a house that costs \$650,000. Gerry takes out a loan for the rest of the cost of the house. The loan charges 3% interest compounded monthly amortized over some period of time. Gerry's monthly mortgage payments are \$2883.91. Over how many years is Gerry's loan amortized?

   ANSWER: _____years

   (c) Eight and a half years after taking out the loan and making monthly payments, Gerry inherits a large sum of money and wishes to pay off the loan. What is Gerry's payoff amount?

   ANSWER: \$_____