MATH 111 B
Exam I
Autumn 2018

Name _____

Student ID #_____ Section _____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- Check that your exam contains **three** problems.
- You are allowed to use a TI-30XIIS calculator, a ruler, and a 3-inch-by-5-inch note card containing hand-written notes. All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- You must show your work. Clearly label lines and points that you are using and show all calculations. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- When rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the back of the last page and **indicate that you have done so**. If you still need more room, ask your TA for an extra page to staple to your exam.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.

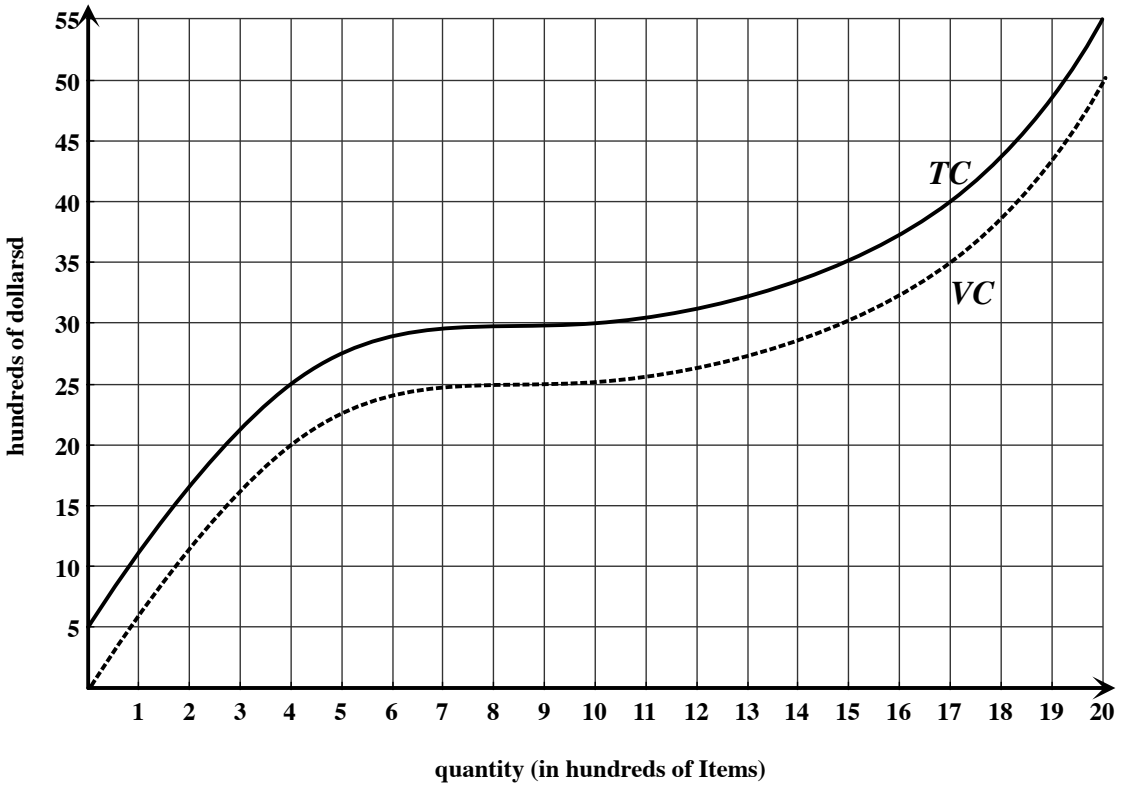
GOOD LUCK!

Suppose you produce and sell Things. The following table summarizes the terms we've learned so far relating to revenue and cost. Assume you are given a graph of total cost $TC(q)$ and total revenue $TR(q)$ for producing and selling q Things.

Term	**Definition**	**Related equations and formulas**	**Graphical Interpretation**
total cost $TC(q)$	the total amount you spend to produce q Things	$TC(q) = VC(q) + FC$	—
variable cost $VC(q)$	the money you spend to produce q Things without including fixed costs	$VC(q) = TC(q) - FC$	the graph of VC has the same shape as TC and goes through the origin
fixed cost FC	the money you must spend even if you produce 0 Things; also known as *overhead*	$FC = TC(q) - VC(q)$ $FC = TC(0)$	the vertical distance between the TC and VC graphs OR the "y"-intercept of the TC graph
average cost $AC(q)$	total cost averaged over the number of Things produced	$AC(q) = \frac{TC(q)}{q}$	the slope of the diagonal line through the TC graph at q
average variable cost $AVC(q)$	variable cost averaged over the number of Things produced	$AVC(q) = \frac{VC(q)}{q}$	the slope of the diagonal line through the VC graph at q
breakeven price BEP	the smallest value of average cost	—	the slope of the least steep diagonal line that intersects the TC graph
shutdown price SDP	the smallest value of average variable cost	—	the slope of the least steep diagonal line that intersects the VC graph
marginal cost $MC(q)$ (see footnote)	the incremental rate of change in TC from q to $q+1$ Things	$MC(q) = \frac{TC(q+1)-TC(q)}{1}$	the slope of the secant line through TC (or VC) at q and $q+1$
total revenue $TR(q)$	the total amount you receive when you sell q Things	—	—
average revenue $AR(q)$	total revenue averaged over the number of Things sold; also known as *price per Thing*	$AR(q) = \frac{TR(q)}{q}$	the slope of the diagonal line through the TR graph at q
marginal revenue $MR(q)$ (see footnote)	the incremental rate of change in TR from q to $q+1$ Things	$MR(q) = \frac{TR(q+1)-TR(q)}{1}$	the slope of the secant line through the TR graph at q and $q+1$
profit $P(q)$	the money you are left with after subtracting total cost from total revenue	$P(q) = TR(q) - TC(q)$	the vertical distance between TR and TC (when $TR > TC$)

NOTE: If q is measured in hundreds or thousands of Things, the definitions, formulas, and graphical interpretations of marginal revenue and marginal cost must be adjusted appropriately.

1. (17 points) The graph below shows **total cost** and **variable cost** for producing Items.



(a) Compute the **marginal cost** to produce 6 hundred Items.

ANSWER: _____dollars per Item

(b) Find the shutdown price.

ANSWER: _____dollars per Item

(c) Find the largest interval, starting at $q = 1$ hundred Items, on which **average variable cost** is *at least* \$2.50 per Item.

ANSWER: from $q = 1$ to $q =$_____hundred Items

(d) You sell Items for \$$p$ each. If you sell 1000 Items (10 hundred Items), **total revenue** is \$4000 (40 hundred dollars).

i. Sketch and label the graph of **total revenue** on the axes above and compute the value of p.

ANSWER: Items sell for _____dollars each

ii. What quantity maximizes **profit**?

ANSWER: $q =$_____hundred Items

2. (16 points) A Purple car and a Green car travel along the same road, beginning from the same place at time $t = 0$ minutes.

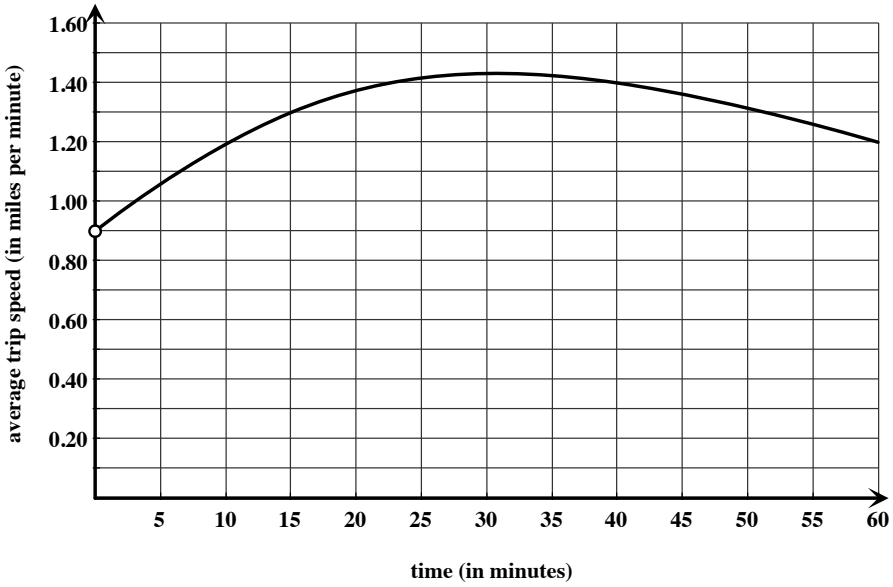
   (a) Let $P(t)$ represent the distance traveled by the Purple car in t minutes and let $G(t)$ represent the distance traveled by the Green car in t minutes.

   Translate each of the following into **functional notation**:

   i. During the 5-minute interval starting at time t, the Purple car's average speed is 0.9 miles per minute.

   ii. At time t, the Green car is 15 miles ahead of the Purple car.

   Here is the graph of **average trip speed** for the Purple car.



   (b) What is the highest value of the Purple car's **average trip speed**?

   ANSWER: _____miles per minute

   (c) **How far** does the Purple car travel in the first 60 minutes?

   ANSWER: _____miles

   (d) What is the Purple car's **average speed** from $t = 15$ to $t = 40$?

   ANSWER: _____miles per minute

3. (17 points) You sell Things for \$32 each.
   The **profit** function for selling Things is **linear**.
   If you sell 100 Things, your profit is \$420. If you sell 200 Things, your profit is \$1120.

   (a) Find the formula for $R(x)$, the **total revenue** (in dollars) for selling x Things.

   ANSWER: $R(x) =$_____

   (b) Find the formula for $P(x)$, the **profit** (in dollars) to produce x Things.

   ANSWER: $P(x) =$_____

   (c) What is the **total cost** to produce 150 Things?

   ANSWER: _____dollars

   (d) What is the value of your **fixed cost**?

   ANSWER: $FC =$_____dollars

   (e) Find the smallest production level at which you do not take a loss.

   ANSWER: $x =$_____Things