

## Supplement Graphs Review

In this review, I will talk through the various types of graphs you have seen in class and in the homework. I will discuss common questions and translation of problems.

### Total Amount (*i.e.* Distance or Gallons or Dollars) versus Time Graphs:

See Sup. 1-3 Problems 1, 2, and 4; Sup. 4 Problems 2 and 4; Sup. 5 Problems 1-4; Sup. 6-7 Problems 1-3; Sup. 8-9 Problems 1-2.

1. A **slope** on such a graph is a **rate**. The units will be the  $y$ -units over the  $x$ -units.

2. Special terms:

graphically	distance	water amount	$TC(q)$	$VC(q)$	$TR(q)$
slope of diagonal line	$ATS(t)$	NO NAME	$AC(q)$	$AVC(q)$	$AR(q)$
slope of secant	$AS(t)$	rate of flow	$MC(q)$	also $MC(q)$	$MR(q)$

3. If you are asked to **find a slope or a rate**, then you put the ruler on the page, draw the line and use any two points to find the slope. There are many examples in the homework. Here are a few: Sup. 1-3 Problem 1(f), Sup. 1-3 Problem 3(a)(b)(c)(d), Sup. 4 Problem 2(a), Sup. 4 Problem 4(d)(e), and Sup. 8-9 Problem 1(a)(c)(d). (again there are many, many other examples in the homework).

4. If you are **given a slope or a rate and need to find a time or interval**, then you draw a reference line! That is, draw a line with the given slope and use that line for reference. Then you slide the ruler parallel to the reference line until it forms a secant on the graph with the desired interval length/location. For examples see: Sup. 4 Problem 2(b)(c), Sup. 5 Problem 4(c), and Sup. 8-9 Problem 2(f)(g) (among many others).

5. If the interval is small on the graph, draw the lines as a tangent (for example  $\frac{TG(1.1)-TG(1)}{0.1}$  from the homework or whenever you compute  $MR(q)$  or  $MC(q)$ ).

6. The **vertical gap** (straight up and down) between two graphs is the difference between their values.

- In the reservoir problems: If water IN is higher than water OUT, then there is a surplus of water in the reservoir. If water OUT is higher than water IN, then there is a shortage. In those problems, you are often asked to find the greatest shortage, that would be when OUT is above IN by the largest amount.
- In the cars in and out of a parking garage problem: If IN is higher than OUT, then more cars are in the garage than when we started keeping track. If OUT is higher than IN, then fewer cars are in the garage than when we started keeping track.
- In the TR/TC problems: Profit is the vertical gap between TR and TC. If TR is above TC, then profit is positive and if TR is below TC, then profit is negative. The largest vertical gap when TR is above TC is the maximum profit.

NOTE: We saw in studying TR/TC, that **the largest vertical gap always occurs at a location when the two graphs have the same tangent slopes**. So we can slide our ruler between the two graphs and make sure they have the same slope. At this value, the gap is maximum! You can use this method whenever you are finding largest gaps in any of these scenarios.

**Increments or Rates Graphs:** See Sup. 1-3 Problems 3; Sup. 4 Problems 1 and 3; Sup. 8-9 Problem 3.

- When you see a graph has incremental or rate information first **read the description carefully!** If the  $y$ -axis says ‘change in value’ or ‘inches/month’ or ‘dollar/item’ or ‘miles/hour’, then you know that are dealing with incremental or rate information. In these problems, you typically are **not using/sliding/rotating your ruler**, when given incremental or rate graphs you already have the rates! So you are just reading off values. Once you realize you are dealing with an incremental or rate graph, I always suggest you read off a few values and make a table (just to make sure you understand what each point means). Here is a discussion of every such homework question:

- Sup. 1-3 Problem 3: You are given a graph of the values of  $ATS$ . First remember that  $ATS(t) = \frac{D(t)}{t}$ . So you will need to read off points and use the definition. Let’s make a table of values to get a sense of what is going on:

Time	Value of ATS on graph	This Means	Which Gives Dist
5 min	1.9 mpm	$1.9 = \frac{D(5)}{5}$	$D(5) = 1.9 \cdot 5 = 9.5$ miles
10 min	1.81 mpm	$1.81 = \frac{D(10)}{10}$	$D(10) = 1.81 \cdot 10 = 18.1$ miles

and so on...

Now it should be easier to answer the given questions.

- Supp. 4 Problem 1: The description tells you each dot represents the incremental rate (inches/month) for the previous 3 months. Remember rate =  $\frac{\text{change in height}}{\text{change in time}}$ . Again, let’s make a table of values to get a sense of what is going on:

Interval	Value on graph	This Means	Which Gives Change in Height
0-3 months	0.275 in/mo	$0.275 = \frac{\text{change in height}}{3-0}$	$0.275 \cdot 3 = 0.825$ inches
3-6 months	0.35 in/mo	$0.35 = \frac{\text{change in height}}{6-3}$	$0.35 \cdot 3 = 1.05$ inches

and so on...

Again, now it will be easier to answer questions.

- Supp. 4 Problem 3: The description tells you each dot represents the change in temperature over the next two hours (in degrees). So this is just the change in value (there are no rates here). Let’s make a table and we’ll keep track of total change.

Interval	Value on graph	This Means
0-2 hours	-1 degrees	temp went down 1 degree
2-4 hours	0 degrees	temp did not change
4-6 hours	1 degrees	temp went up 1 degree
6-8 hours	1.5 degrees	temp went up 1.5 degrees
8-10 hours	2 degrees	temp went up 2 degrees

and so on...

- Supp. 8-9 Problem 3: Graphs of  $AC$ ,  $AVC$  and  $MC$  are given. To understand this graph, we need to remember the definitions:  $AC(q) = \frac{TC(q)}{q}$ ,  $AVC(q) = \frac{VC(q)}{q}$  and  $MC(q) = \frac{TC(q+1)-TC(q)}{(q+1)-q}$ . Again, let’s read a few values off the values:

Quantity	$MC$	$MC$ meaning
2	10 \$/item	going from producing 2 to 3 adds \$10 to cost
3	9.1 \$/item	going from producing 3 to 4 adds \$9.1 to cost
4	8.3 \$/item	going from producing 4 to 5 adds \$8.3 to cost

Quantity	$AVC$	$AVC$ meaning	$AC$	$AC$ meaning
28	3.6 \$/item	$3.6 = \frac{VC(28)}{28}$	11 \$/item	$11 = \frac{TC(28)}{28}$
29	3.5 \$/item	$3.5 = \frac{VC(29)}{29}$	11 \$/item	$10.6 = \frac{TC(29)}{29}$

So you can multiply to get  $VC(28) = 3.6 \cdot 28$  or to get  $TC(29) = 10.6 \cdot 29$  (you will have to do this in homework).

## Business Terms and Graphs - When given TR, TC, and VC

1. First remember that  $FC = TC(0)$ . And if you are given market price per item, then  $TR$  is just a line through the origin (the slope is the market price). Also note that  $TC(q) = FC + VC(q)$ , so  $TC(q)$  and  $VC(q)$  are the same graphs but with  $TC$  shifted up by  $FC$ .
2. We discussed that  $P(q) = \text{profit} = TR(q) - TC(q) =$  ‘the vertical gap between  $TR$  and  $TC$ ’ (it is positive if  $TR > TC$  and negative if  $TR < TC$ ).
3. Know how to compute from the graphs as follows:
  - $MR(q) =$  ‘slope of the “secant” line to  $TR$  from  $q$  to the next item’ (this typically looks like a tangent line).
  - $MC(q) =$  ‘slope of the “secant” line to  $TC$  (or  $VC$  from  $q$  to the next item’ (again, this looks like a tangent line).
  - $AR(q) =$  average revenue per item = ‘slope of the diagonal line through the  $TR$  graph’
  - $AC(q) =$  average cost per item = ‘slope of the diagonal line to the  $TC$  graph’ (average cost including fixed costs).
  - $AVC(q) =$  average variable cost per item = ‘slope of the diagonal line to the  $VC$  graph’ (average cost without fixed costs).

UNITS: Remember that all of these are slopes. In all our examples, slope is in units of dollars/item. Even if the  $x$  and  $y$  units are in hundreds or thousands, the slope remains dollar/items as we discussed in class (remember the cancelling of the hundreds/thousands). So **MR, MC, AR, AC, and AVC are always in units of dollars/item.**

4. Here are the most common applications we have encountered:
  - *Find quantities when the profit is zero.* This is the same as asking when is  $TR = TC$ . So you need to find the values of  $q$  at which  $TR$  and  $TC$  intersect.
  - *Find the quantity at which profit is maximum.* We discussed three methods:
    - (a) METHOD 1: Find the location of the largest gap when  $TR$  is above  $TC$ . (Hard to be precise with this method, but it works).
    - (b) METHOD 2: Find when  $MR = MC$ . (Good if you have a table of  $MR/MC$  values or if you have  $MR/MC$  graphs).
    - (c) METHOD 2’: Match Slopes! That is, find when the slope of tangent to  $TR$  is the same as the slope of the tangent to  $TC$ . (this is just the graphical way of saying  $MR = MC$ , but it is the easiest and most precise method to use when given graphs of  $TR$  and  $TC$ ).
  - *Find the maximum profit.* This is asking about the same quantity you get from the previous question above, but it wants the actual dollar amount. That is, we want you to estimate the size of the gap. It’s best to just eyeball the values of  $TR$  and  $TC$  and then subtract.
  - *Find the Breakeven Price (BEP).* Fix your ruler at the origin and rotate to the lowest diagonal line that touches the  $TC$  graph. Find the slope of this line! (Get two points and find the slope). Remember, this slope represents a transitional market price. If the market price is above the BEP, then it is *possible* to make a positive profit. If the market price is below the BEP, then it is NOT possible to make a positive profit.
  - *Find the Shutdown Price (SDP).* Fix your ruler at the origin and rotate to the lowest diagonal line that touches the  $VC$  graph. Again, find the slope of this line! Recall: If the market price is above the SDP, then it is *possible* to recover some of your fixed costs (stay open). If the market price is below the SDP, then you are going to lose more money than your fixed costs no matter how many items you sell (shut down).

## Business Terms and Graphs - When given graphs of MR, MC, AC, and AVC

1. It is fairly common to see graph of rates in business examples. The last example in the homework from Supplement 8-9 gives you such an example. I already discussed this graph earlier in this review when I was talking about increment or rate graphs, but there are a few more specific notes I want to make about this particular graph. Remember these are graphs of rates, so **put your ruler away**, we are not finding slopes on this graph. These are already rates, we will just be reading off values!

2. Let's discuss the some common business questions and how they can be answered from this graph.

- *Find the quantity at which profit is maximum.*

We are NOT talking about finding gaps any more (and not sliding the ruler). You don't have a graph of TR and TC, you have a graph of MR and MC! The  $y$ -values of  $MR$  and  $MC$  are values of slopes that you would find if you had  $TR$  and  $TC$ .

Use METHOD 2: Find when  $MR = MC$ . That is find the quantity when the  $MR$  and  $MC$  graphs intersect! That should be quick and easy, just read off quantity  $q$  at which they intersection.

- *Find the Breakeven Price (BEP).*

We are NOT finding the lowest diagonal slope to  $TC$  with our ruler (because we aren't given  $TC$ ). What we need to remember is that the  **$y$ -coordinates on the AC graph are the values of the slopes of the diagonals to the TC graph** and we just want the lowest  $AC$  value.

So  $BEP =$  'the lowest  $y$ -coordinate value on the  $AC$  graph'.

Also, think back to when we drew the lowest diagonal to the  $TC$  graph, remember that line also looked like a tangent. That means it is a value of  $MC$ . So you can say this in another way.

$BEP =$  'the  $y$ -coordinate value where the  $AC$  and  $MC$  graphs intersect'.

- *Find the Shutdown Price (SDP).*

As before, remember the  **$y$ -coordinates on the AVC graph are the values of the slopes of the diagonals to the VC graph** and we just want the lowest  $AVC$  value.

So  $SDP =$  'the lowest  $y$ -coordinate value on the  $AVC$  graph'.

And this can be said another way (because the lowest diagonal to  $VC$  is also a tangent):

$SDP =$  'the  $y$ -coordinate value where the  $AVC$  and  $MC$  graphs intersect'.