Section 6.1-6.5 Overview/Preview

We will discuss the topics of interest bearing accounts over the next several lectures, but I wanted to give you all the formulas up front so that you had them in one convenient place. First some notation:

- $P = PV$ = present value = value at the beginning of the time interval in question.
- $F = FV$ = future value = value at the end of the time interval in question.
- r = annual interest rate (expressed as a decimal).
- m = number of compounding periods in a year.
- t = time in years.
- R = deposit or payment for an annuity or mortgage (one payment each compounding period).
- $i = \frac{r}{m}$ = rate divided by the number of compounding periods.
- $n = mt$ = years times number of compounding in a year (*i.e.* total compounding periods).

Formulas:

Simple Interest (Section 6.1)	$F = P(1+rt)$
Compounding m times a year (Section 6.2)	$F = P\left(1+\frac{r}{m}\right)^{mt} = P(1+i)^n$
Continuous Compounding (Section 6.2)	$F = Pe^{rt}$
Ordinary Annuity Future Value (Sections 6.3)	$F = R\left[\frac{(1+i)^n-1}{i}\right]$
Ordinary Annuity Present Value (Sections 6.4-6.5)	$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$
Annuity Due Future Value (Section 6.3)	$F = R\left[\frac{(1+i)^n-1}{i}\right](1+i)$
Annuity Due Present Value (Sections 6.4-6.5)	$P = R\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$

Lump Sum Account Examples (in each example, the values at $t = 1$, 2, and 20 are shown):

1. Simple Interest: Suppose we start with \$1000 in a CD account that pays 2% simple interest annually. That means that each year the account will add $0.02 \cdot 1000 = \$20$.

Years:	0	1	2	$\cdots$	20
Formula:	$1000(1+0.2(0))$	$1000(1+0.2(1))$	$1000(1+0.2(2))$	$\cdots$	$1000(1+0.02(20))$
Value:	\$1000	\$1020	\$1040	$\cdots$	\$1400

2. Discrete Compounding: Suppose we start with \$1000 in an account that pays 2% interest, compounded quarterly.

Years:	0	1	2	$\cdots$	20
Formula:	1000	$1000\left(1+\frac{0.02}{4}\right)^{4(1)}$	$1000\left(1+\frac{0.02}{4}\right)^{4(2)}$	$\cdots$	$1000\left(1+\frac{0.02}{4}\right)^{4(20)}$
Value:	\$1000	\$1020.15	\$1040.71	$\cdots$	\$1490.34

3. Continuous Compounding: Suppose we start with \$1000 in an account that pays 2% interest, compounded continuously.

Years:	0	1	2	$\cdots$	20
Formula:	1000	$1000e^{0.02(1)}$	$1000e^{0.02(2)}$	$\cdots$	$1000e^{0.02(20)}$
Value:	\$1000	\$1020.20	\$1040.81	$\cdots$	\$1491.82

Annuity Examples (again, I give values at t=1, 2, and 20):
For all the examples below the compounding and payments/deposits are monthly (m=12) and the account has an annual interest rate of 2% compounded monthly (so $r = 0.02$).
For simplicity, we compute $i = \frac{r}{m} = \frac{0.02}{12} = 0.001666667$.

1. Ordinary Annuity:

   (a) Future Value: You start putting \$100 into the account at the END of each month.

Years:	1	2	$\cdots$	20
Formula:	$100\left[\frac{(1+i)^{12(1)}-1}{i}\right]$	$100\left[\frac{(1+i)^{12(2)}-1}{i}\right]$	$\cdots$	$100\left[\frac{(1+i)^{12(20)}-1}{i}\right]$
Value:	\$1211.06	\$2446.57	$\cdots$	\$29,479.68

   (b) Present Value: You want \$100 deposited into your checking account from your retirement account at the END of each month. Here is how much you would have to start with so you would have enough money in the account to receive these payments for the indicated number of years.

Years:	1	2	$\cdots$	20
Formula:	$100\left[\frac{1-(1+i)^{-12(1)}}{i}\right]$	$100\left[1-\frac{(1+i)^{-12(2)}}{i}\right]$	$\cdots$	$100\left[1-\frac{(1+i)^{-12(20)}}{i}\right]$
Value:	\$1187.10	\$2350.71	$\cdots$	\$19,767.40

2. Annuity Due:

   (a) Future Value: You start putting \$100 into the account at the BEGINNING of each month.

Years:	1	2	$\cdots$	20
Formula:	$100\left[\frac{(1+i)^{12(1)}-1}{i}\right](1+i)$	$100\left[\frac{(1+i)^{12(2)}-1}{i}\right](1+i)$	$\cdots$	$100\left[\frac{(1+i)^{12(20)}-1}{i}\right](1+i)$
Value:	\$1213.08	\$2450.65	$\cdots$	\$29,528.82

   (b) Present Value: You want \$100 deposited into your checking account from your retirement account at the BEGINNING of each month. Here is how much you would have to start with so you would have enough money in the account to receive these payments for the indicated number of years.

Years:	1	2	$\cdots$	20
Formula:	$100\left[\frac{1-(1+i)^{-12(1)}}{i}\right](1+i)$	$100\left[1-\frac{(1+i)^{-12(2)}}{i}\right](1+i)$	$\cdots$	$100\left[1-\frac{(1+i)^{-12(20)}}{i}\right](1+i)$
Value:	\$1189.08	\$2354.63	$\cdots$	\$19,800.35