Exam 2 Review

Exam 2 covers 1.6, 2.1-2.3, 1.5, 4.1-4.2, and 5.1-5.3. You should know how to do all the homework problems from these sections and you should practice your understanding on several old exams in the exam archive. Make sure you are working through the problems on your own (without looking at solutions or getting help). Put yourself in an exam like setting as you are studying.

Here is a brief summary of things we have done:

- **Finding the equation of a line**: Get two points, find the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and write $y = m(x - x_1) + y_1$.

- **Supply-Demand Problems**: Market equilibrium occurs where supply and demand intersect (so you need to solve a system of equations). If price is above market equilibrium, then there is a surplus. And if price is below market equilibrium, then there is a shortage. Know how to solve the system to find market equilibrium and understand the basics of this scenario.

- **Know Function Definitions and How to Use Functional Notation**: You should know how to go from price to TR, how to go from AC to TC, how to go from AVC to VC, how to go from TR/TC to MR/MC, how to go from Distance to ATS, etc.... In other words you need to know the function definitions and how to use them.

- **Remember the Standard Applications We Have Been Discussing All Term**: Maximum profit, break even quantity, break even price (BEP), shut down price (SDP), etc...

- **Know How to Find and Interpret the Vertex of a Parabola**: Once you have a quadratic function $y = ax^2 + bx + c$, then you should know that the vertex occurs at $x = -\frac{b}{(2a)}$. You should also know that if $a > 0$, then the parabola opens upward (so the vertex is a minimum) and if the $a < 0$, then the parabola opens downward (so the vertex is a maximum). You need to be able to interpret what you have to answer various questions.

- **Know How to Solve Equations involving Quadratics**: You need to be comfortable using the quadratic formula. If you are able to get one side to zero and end up with $ax^2 + bx + c = 0$ (so there is only one variable x, and the expression on one side is a quadratic and the other side is zero!), then you can solve by using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{(2a)}$. You need to be able to interpret what you get to answer various questions.

- **Be able to Solve a System of Equations**: Solve for one variable in one equation and substitute into the other equation.

- **Know How to Do Linear Programming**: You need to be able to read information to get constraints and an objective. You need to be able to graph inequalities. You need to be able to **solve a system of equations** to get corners. And you need know how to finish the problem (evaluate the objective at each of the corners).

- **Know the Basics of Working with Exponentials and Logarithms**: Understand what we did in 5.1, 5.2, and 5.3. Evaluating exponentials, working with powers and roots, working with exponentials and logarithms, solving, and using exponential models.

Examples of problems of each type from homework are on the next page. (You need to know all the homework, I just randomly grabbed examples of each type of problem and put them on this review, so that you could remember what those types of problems look like).

- **Finding the equation of a line**.

  1. HW 1.6/5: "The total cost function is linear, and the total cost for 90 sweatshirts is $4770, whereas the total cost for 280 sweatshirts is $8380."'

  2. HW 1.6/9: "Assuming that the demand function is linear, write its equation."

  3. HW 1.6/11: "Find the linear functions for supply and demand."

  4. HW 2.3(p2)/2: "You charge $24 per item for an order of 1 item and $18 per item for an order of 13 items. If price, p, is a linear function of quantity, q, then find the formula for price."

- **Supply-Demand Problems**:

  1. HW 1.6/7-11: "Is there are shortage or surplus when $p = \$20$? (and by how much)"

  2. HW 1.6/7-11: "Find the market equilibrium"

  3. HW 2.3(pt2)/4-6: "If $p^2 + 8q = 1200$ is the demand and $300 - p^2 + 2q = 0$ is the supply, then find the quantity and price at market equilibrium"

  4. HW 1.5/7: "If supply and demand are given by $p - q = 10$ and $q(2p - 10) = 300$, what is the market equilibrium."

- **Know Function Definitions and How to Use Functional Notation**:

  1. HW 2.2/1 and 2: "Find the average rate of change of $y = 2 + 5x + 0.5x^2$ between $x = 4$ and $x = 6$."

  2. HW 2.2/4-6: "If $f(x) = 5 + x + x^2$, find $\frac{f(x+h)-f(x)}{h}$ and simplify."

  3. HW 2.2/8-9: "If $D(t) = t - 0.025t^2$, find the formula for $ATS(t)$."

  4. HW 2.2/8-9: "If $D(t) = t - 0.025t^2$, find the formula for the average speed over a 5-min interval starting at t minutes. That is find $AS(t) = \frac{D(t+5)-D(t)}{5}$."

  5. HW 2.3(p1)/5: "If price is given by $p = 140 - 0.80x$, then find the formula for revenue $R(x) = $??" Then you are asked to find the price that corresponds to maximum revenue (which is a vertex question).

  6. HW 2.3(p1)/7: "If $VC(q) = \frac{1}{30}q^3 - \frac{3}{10}q^2 + q$ and $FC = 0.8$, then find formulas for $TC(q)$, $AC(q)$, and $AVC(q)$."

  7. HW 2.3(p1)/8: "If $TR(q) = -0.25q^2 + 30q$ and $TC(q) = 17.5q + 100$, give formulas for $MR(q)$, $MC(q)$, $AR(q)$ and $AC(q)$."

  8. HW 2.3(p2)/2: "If price is given by $p = 24.5 - 0.5q$, then find formulas for $TR(q)$ and $MR(q)$."

  9. HW 2.3(p2)/2: "If the average cost per item is a constant $5 and fixed cost is $100, then give the formula for $TC(q)$."

  10. HW 2.3(p2)/3: "If $AVC(x) = \frac{4}{9}x + 333$, $FC = 54400$ and price is $p = 2065 - \frac{5}{9}x$, then find formula for $TR(q)$, $VC(q)$, $TC(q)$ and profit."

- **Remember the Standard Applications We Have Been Discussing All Term**:

  1. HW 2.3(p1)/1-3: "If cost and revenue are given by $C(x) = 5600 + 80x + x^2$ and $R(x) = 230x$, find the break-even points." (this also involves solving a quadratic).

  2. HW 2.3(p1)/7: "If $AVC(q) = \frac{1}{30}q^2 - \frac{3}{10}q + 1$, then what is the shutdown price (SDP)?" (This is a vertex question).

  3. HW 2.3(p2)/1: "If $MC(q) = 5q + 3$ and $AC(q) = 2.5q + 3 + \frac{15}{q}$, find the break even price (BEP)?" (You will need to solve $MC = AC$ which will require the quadratic formula).

- **Know How to Find and Interpret the Vertex of a Parabola**:

  1. HW 2.2/7: "If profit is $P(x) = 18x - 0.1x^2 - 50$ dollars, find the quantity that maximizes profit."

2. HW 2.2/10: "If $f(x) = x^2 - 4x + 16$ and $g(x) = -0.5x^2 + 4x + 10$, find the longest interval over which $f(x)$ and $g(x)$ are both increasing."

3. HW 2.3(p1)/4: "Find the maximum value of the revenue function $R(x) = 381x - 0.9x^2$."

4. HW 2.3(p1)/8: "If $TR(q) = -0.25q^2 + 30q$ and $TC(q) = 17.5q + 100$, what quantity maximizes profit?"

5. HW 2.3(p1)/9: "What is the largest value of total revenue?"

6. HW 2.3(p1)/9: "Give the longest interval on which total revenue and profit are both increasing."

7. HW 2.3(p2)/3: "Find the maximum profit and determine the corresponding price per unit that yields this profit."

- **Know How to Solve Equations involving Quadratics**:

  1. HW 2.1/1-4: "Solve $\frac{w^2}{8} - \frac{w}{2} - 4 = 0$."

  2. HW 2.1/7: "If profit is $P(x) = 135x - 100 - x^2$, what quantity will yield a profit of \$3940?"

  3. HW 2.2/10: "If $g(x) = -0.5x^2 + 4x + 10$, find the interval over which $g(x)$ is greater than or equal to 15."

  4. HW 2.3(p1)/7: "For what range of quantities is $AVC(q)$ at most \$0.55 per bag?"

  5. HW 2.3(p1)/9: "Give the largest quantity at which profit is non negative." (this is asking you to find when profit $= 0$ and give the larger of the two answers).

- **Be able to Solve a System of Equations**:

  1. HW 1.5/1-3: "Solve the system given by (i) $3x + 9y = -3$ and (ii) $2x - 3y = 16$."

  2. HW 1.5/4: "Investments are made into two accounts (one at 10% annual interest and one at 11% annual interest). The total amount invested is \$142,000. The interest earned in the first year is \$14,800. How much was initially invested in each account?"

  3. HW 1.5/8: "If supply is $2p - q - 20 = 0$ and demand is $(p + 20)(q + 10) = 6300$, find the market equilibrium."

- **Know How to Do Linear Programming**:

  1. HW 4.1/2: "Graph the overlapping region given by the constraints $3x + 9y \geq 18$, $2x + 4y \geq 10$, $9x + 3y \geq 15$, $x \geq 0$ and $y \geq 0$."

  2. HW 4.1/6-9: These are word problems (they are longer so I won't copy them here), you need to translate to inequalities and graph.

  3. HW 4.2/3: "Maximize $f(x, y) = 4x + 5y$ subject to the constraints: $x + y \leq 7$, $2x + y \leq 10$, and $y \leq 6$."

  4. HW 4.2/4-7: Again, more word problems to practice.

- **Know the Basics of Working with Exponentials and Logarithms**:

  1. HW 5.1&5.2/1: "Evaluate $S(x) = 1400(1.02)^{4x}$ at $x = 8$."

  2. HW 5.1&5.2/10: "Solve $8e^{2t-5} = 24$."

  3. HW 5.3/1: "Solve $4^{3x} = 56364$."

  4. HW 5.3/3: "Solve $83 = 100 - 100e^{-0.04x}$."

  5. HW 5.3/8: "The population in a country is 100000 in 1998 and 110585 in 2008. Let $t = 0$ correspond to 1998, and $t = 10$ correspond to 2008. You are told that the population grows according to an exponential function of the form $y = P_0 e^{ht}$. Find P_0 and h using the given information, then use the formula to give the population in 2023."

  6. HW 5.3/14: "Solve $100 = 500(0.03)^{(0.7^t)}$."