Durable Anchors

Making Mathematics Memorable Enough to Matter

Andy Loveless

Preface: Why Memorable Teaching Matters More Than Ever

I should start by saying that I do not claim to have teaching "figured out." That feels important to say early. I have taught over 20,000 freshman and sophomore math students at UW over the last 20 years. I've won the biggest teaching awards on my campus. I've made hundreds of review sheets, answered a frightening number of emails, built too many websites, recorded mountains of videos, and told many bad puns in lecture.

But the truth is that I have mostly tried to survive day-to-day and year-to-year. That is what heavy teaching loads do to a person, but along the way I think I developed a few key principles that mattered to me and my students. They are ideas and principles that are rarely discussed, even though many teachers likely use them. I want to bring them more centrally into the light. I hope something here resonates with you, and I hope this encourages you to share your own stories and methods, because I want to hear them.

What I have learned, after many years of teaching large freshman and sophomore math courses, is that students need something to hold onto. They need clear explanations, good examples, practice problems, review sheets, office hours, honest advice, and help when they get stuck. They need all of that.

But they also need moments that stay with them.

A story. A picture. A bad joke. A song. A graph. A Pringles chip. A tennis ball flying through the air. A cat in a Desmos animation. A campus sculpture they have walked past a hundred times but never really looked at before. Exam examples where they throw water balloons at their instructor. And many more.

I have decided to call these moments *durable anchors*.

A durable anchor is not a trick. It is not entertainment sprinkled on top of the real class. It is a way of giving an idea somewhere to live. If a student can remember the tennis ball, they may be able to remember the graph. If they remember the graph, they may be able to remember velocity. If they remember velocity, maybe the formula has a fighting chance.

That is the basic hope.

The First Two Weeks

In every large math class I teach, there is a battle in the first two weeks. The battle is not with the students. At least I try very hard not to make it that. The battle is with years of students being told, or telling themselves, that they are bad at math.

I hear it all the time. Students say it. Parents say it. Sometimes people say it almost proudly, as if being bad at math is a badge of honor. I understand where it comes from, but I do not accept it as the final word.

I firmly believe that every person can learn math, wants to learn math at some level, and needs math. But some have not yet received the invitation they need to be hooked into the subject.

Some students need more practice. Some need more time. Some need a different picture, a different story, a different explanation, or a different starting point. My job is to keep looking for that approach.

They have to keep trying, and I have to keep trying. We fail if either one of us gives up.

That is why the first two weeks matter so much. I have to convince students that they can understand the material if they work hard. I also have to earn their trust so they believe I will keep looking for another explanation until something clicks.

Once they believe they can do it, and once they believe I am not their opponent, the class changes. Then we can get to work.

Why the Memorable Moments Matter

The first thing I often tell my classes is that I am not really "Loveless." I am married with kids. It is a bad joke, but it works well enough. It tells students that I am willing to make fun of myself. It wakes them up a little. It also lowers the temperature in the room.

Sometimes my examples involve my kids, pets, broccoli, or whatever object seems useful at the moment. Sometimes there is a fun visual, often with my cats in the background. Every day, I move around the room. It is fun to stand at the back while students work a problem; they keep peeking around at me. Sometimes I bring 3D prints to class or a slinky. I always have a tennis ball to throw around.

The goal is not to make every class funny. The goal is to make the class feel human enough that students are willing to keep thinking. That matters. A lot.

And it is not just about the three 50-minute sessions in class. My approach continues on the discussion board or in short supplemental, not-required, fun videos after class. I also make fun videos visualizing exam problems and explaining why they are interesting after each exam. I am looking for many ways to invite students back into the material.

Students can find answers everywhere now. They can look up formulas, watch videos, ask a computer, or search for a worked example. But finding an answer is not the same as caring about the question.

It is not the same as building confidence. It is not the same as learning how to look at the world and say:

> There is a pattern here. There is a story here. I can understand this.

That is what I want for my students.

I want them to pass the exam, of course. I want them to do the homework, learn the methods, and move forward in their majors. The practical side matters. At UW, the pressure is real and sometimes terrible, and many students are carrying a lot.

But I also want them to leave with something more than a list of techniques.

I want them to remember that math can explain motion, shape, fairness, sound, games, light, growth, and the strange little problems that show up in ordinary life. I want them to have anchors.

The Jokes Are Bad, But They Work

I should probably admit something else: I like puns. This is not a strength. There are probably limits to how much a person should do this, and I have likely crossed several of them.

I once wrote a poem called "Ev-Riemann Should Learn Calculus," which tried to put as many calculus terms as possible into ordinary sentences. I have made a 67-second video on why all numbers are worth exploring, even 67. I made a poem using every letter of the Greek alphabet to make fun of brainrot words. When we solve for the letter p, I always end by saying, "I got a p."

So many bad jokes. But it keeps students listening.

I revel in the moments when they find my mistakes. Students notice this. They often say that I laugh a lot in class. I like that. I want the room to feel like a place where people can laugh and still work hard.

The jokes are not there because mathematics is unserious. The jokes are there because mathematics is serious enough that students need permission to breathe while learning it.

A bad pun can do a surprising amount of work. It can wake up a sleepy room. It can make a formula less intimidating. It can give students a phrase to remember later. It can also show students that the person at the front of the room is not trying to be above them. I tell students I am proving that you do not have to be smart to do math. Look, I am doing it.

I am very happy to be ridiculous if it helps them stay with the subject.

That does not mean every joke works. Many do not. Some deserve the silence they receive. I tell students in those moments that I will try the same joke in a week and see if it gets the same reaction. But even then, the failure is useful. It reminds students that trying something awkward is allowed.

That is an important lesson for anyone learning math.

Gateway Courses Are Not Disposable

Large calculus and precalculus courses are sometimes treated as courses students just need to get through.

I understand why. These courses serve many majors. They have hundreds of students, common exams, shared expectations, grading logistics, pressure, complaints, and a lot of moving parts. They are hard to teach well and easy to underestimate. But these courses matter enormously.

For many students, a gateway math course is where they decide what mathematics is. It is where they decide whether they belong near it. It is where they decide whether struggle means failure. It is where they decide whether quantitative thinking can be part of their own story. That is a big responsibility.

These courses are not sorting machines. They are not prerequisite delivery systems. They are not obstacles on the way to something more important. If that is how we think at UW, then we are missing a huge opportunity. These courses are one of the places where students learn what learning feels like.

At UW, we can lead here. We can make these courses rigorous and humane, practical and curious, structured and alive. We can help students succeed on exams while also showing them that mathematics is a way of seeing. That work takes time.

It takes examples, visuals, review materials, websites, office hours, videos, emails, and a lot of small acts of explanation. It takes looking for the second, third, fourth, and fifth way to explain something. And it takes departments and administrators that value the teachers giving the time and dedication to find the extra explanation, not just replacing those teachers with the next cheapest option.

We as a school need to be committed to this. None of us should give up on this opportunity and responsibility.

What This Book Is

This book is a collection of durable anchors.

Some are polished. Some are strange. Some started as jokes and became useful. Some started as useful and became jokes. Most came from the same basic teaching problem:

> I was trying to find another way to make an idea click.

The examples are from mathematics because that is the room I have been standing in for 20 years. But I do not think durable anchors belong only to math. Every course should give students something worth carrying with them.

Yes, a class should help students build skills. But it should also help them build stories about themselves and the world.

That is what memorable teaching can do. It can give students a way back into an idea. It can give a formula a place to live.

It can make a lecture, or even thirty seconds of a lecture, matter later. And sometimes, if we are lucky, it can help a student remember that they are not bad at math. They are still learning.

Let's make our courses more memorable. There are lots of ways to do it. I will share many here, and I am sure you have some of your own to share.

I would love to read about your memorable anchors. Let's foster curiosity in many ways and give students tools and stories to draw from as they find their own story.

1 Tennis Balls on Campus

I have used tennis balls in class for years.

If you teach calculus long enough, and if you are willing to look slightly ridiculous in front of a room full of students, a tennis ball becomes a very useful object. You can throw it up and talk about parabolas. You can ask when the velocity is zero. You can ask why the acceleration is still negative even when the ball is moving upward. You can talk about height, slope, tangent lines, and the strange usefulness of turning motion into a graph.

So the tennis ball was not new.

But one day I wondered if I could make it more memorable.

That was the beginning of the problem. Not a theorem. Not a worksheet. Not a grand teaching philosophy. Just a small question:

> What could I do with this tennis ball that students might actually remember?

So I went outside and recorded myself throwing a tennis ball toward the top of George Washington's head.

This is not usually how mathematical research begins. It is also not usually how serious curriculum design is described. But it is, apparently, how some of my better ideas begin.

Image placeholder

Insert image of me throwing the tennis ball near the George Washington sculpture.

At first, I only wanted a short video clip. I wanted students to see that the height of the ball is not really a function of where the ball is horizontally. The useful graph is height versus time. The ball rises, slows down, reaches a highest point, and falls. The height graph tells that story.

But once I started doing the problem, the problem started giving back.

The Classroom Version

In class, I usually start with something much simpler than a campus sculpture.

I bring a tennis ball.

I throw it toward the ceiling.

Then I ask students what they noticed.

They usually say the ball went up and came down, which is not wrong. It is also not calculus yet. So we keep going.

When was the ball moving upward?

When was it moving downward?

When was it highest?

What was happening to the velocity at the top?

What was the acceleration doing while the ball was moving upward?

This last question is one of the useful ones. Many students want the acceleration to be positive while the ball is going up and negative while the ball is going down. That makes sense as an everyday intuition. But the ball is being pulled downward the whole time. Ignoring air resistance, the acceleration is constant:

$$a(t) = -32 \text{ ft/sec}^2.$$

The velocity changes because the acceleration is the slope of the velocity graph. The height changes because the velocity is the slope of the height graph.

That is already a lot of calculus from one throw.

The No-Air-Resistance Model

For the first model, we measure height above my hand. So the ball starts at height

$$h(0) = 0.$$

We assume:

- the ball is thrown nearly vertically,
- the ball is caught at the same height it was thrown,
- air resistance is ignored,

- acceleration due to gravity is approximately 32 ft/sec^2,
- timing is approximate.

Under these assumptions,

$$a(t) = -32.$$

Since velocity is the derivative of height and acceleration is the derivative of velocity,

$$v'(t) = -32.$$

Integrating gives

$$v(t) = v_0 - 32t,$$

where v_0 is the initial upward velocity in feet per second.

Integrating again gives

$$h(t) = v_0 t - 16t^2.$$

This is the height of the ball above my hand after t seconds.

Why the Height Graph Has a Bump

The ball reaches its maximum height when the velocity is zero. In calculus language, this is where

$$h'(t) = v(t) = 0.$$

Since

$$v(t) = v_0 - 32t,$$

the top occurs when

$$v_0 - 32t = 0,$$

so

$$t_{\text{top}} = \frac{v_0}{32}.$$

This is a nice moment in class because the physical picture and the derivative picture match perfectly.

The ball is going up when $v(t) > 0$.

The ball is at the top when $v(t) = 0$.

The ball is going down when $v(t) < 0$.

So the height graph has positive slope, then zero slope, then negative slope.

That is the first derivative test, but with a tennis ball.

I usually call this the "bump" before I call it a critical point. Students know what a bump is. We can add the formal language after the idea has somewhere to live.

Time the Toss. Double It. Square It.

Now suppose the ball is thrown and caught at the same height. If the total time from throw to catch is T, then

$$h(T) = 0.$$

Using

$$h(t) = v_0 t - 16t^2,$$

we get

$$0 = v_0 T - 16T^2.$$

Since $T \neq 0$, this gives

$$v_0 = 16T.$$

The time to the top is therefore

$$t_{\text{top}} = \frac{v_0}{32} = \frac{16T}{32} = \frac{T}{2}.$$

So the top occurs halfway through the flight.

The maximum height above my hand is

$$h_{\max} = h\left(\frac{T}{2}\right).$$

Substituting into the height formula,

$$h_{\max} = v_0\left(\frac{T}{2}\right) - 16\left(\frac{T}{2}\right)^2.$$

Since $v_0 = 16T$,

$$h_{\max} = 16T\left(\frac{T}{2}\right) - 16\left(\frac{T^2}{4}\right).$$

Thus

$$h_{\max} = 8T^2 - 4T^2 = 4T^2.$$

And since

$$4T^2 = (2T)^2,$$

we get the rule:

Time the toss. Double it. Square it.

This gives the maximum height in feet above the thrower's hand, assuming the ball is thrown and caught at the same height and air resistance is ignored.

I am calling this Dr. Loveless's Double It and Square It Rule, which is probably too grand a name for a one-line consequence of introductory calculus, but I am taking what I can get.

George Washington's Head

For my George Washington toss, the measured time from throw to catch was about

$$T = 2.83 \text{ seconds.}$$

The no-air-resistance estimate gives

$$h_{\max} = (2T)^2 = (2 \cdot 2.83)^2.$$

So

$$h_{\max} = 5.66^2 \approx 32.0 \text{ feet.}$$

That is about 32 feet above my hand.

If my hand was about 6 feet above the ground when I threw the ball, then the estimate for the height reached by the ball is

$$32 + 6 \approx 38 \text{ feet.}$$

This lines up nicely with the historical description: roughly a 14-foot bronze figure on a 24-foot pedestal.

Image placeholder

Insert image or still frame showing the tennis ball near the top of George Washington's head.

This is not a good way to do serious surveying.

That is not the point.

The point is that the estimate is quick, physical, memorable, and surprisingly reasonable. It connects a throw, a graph, a derivative, a campus sculpture, and a little bit of history.

That is much more than I expected when I first walked outside with a tennis ball.

The Parabola Is Not the Path

One subtle point is worth saying clearly.

In this example, the ball is moving mostly up and down. The graph

$$h(t) = v_0 t - 16t^2$$

is not a picture of the ball's physical path through space. It is a graph of height versus time.

That distinction matters.

Students often think of parabolas as shapes in space. Sometimes they are. But here the parabola is a story about how the height changes as time passes.

At first the height increases quickly.

Then it increases more slowly.

Then it reaches a maximum.

Then it decreases.

The graph records the motion.

That is a powerful idea. Calculus is not just about drawing curves. It is about using curves to tell stories about change.

What Students Learn From the Throw

This one example opens several important doors.

First, students see that acceleration can be constant even while velocity changes.

Second, they see that velocity can be positive, zero, or negative depending on where the ball is in the motion.

Third, they see that the slope of the height graph is not just an abstract feature of a curve. It has meaning. It is velocity.

Fourth, they see that a maximum occurs when the derivative is zero, at least in this clean example.

Fifth, they see why the first derivative test makes sense. If the height is increasing before the top and decreasing after the top, then the top is a maximum.

In symbols:

$$h'(t) > 0 \quad \text{before the top,}$$

$$h'(t) = 0 \quad \text{at the top,}$$

$$h'(t) < 0 \quad \text{after the top.}$$

That is a lot of calculus for one tennis ball.

The Website Extension

I later built an interactive webpage for this example. The page lets students watch the toss, compare the height and velocity graphs, move through the motion, and read about the campus sculptures.

The page is here:

> https://sites.math.washington.edu/~aloveles/CuriosityLab/OrdinaryThings/sculptures.html

The point of the webpage is not simply to give students another resource. The point is to create more invitations.

A student might come for the video.

Another might come for the graph.

Another might come for the shortcut.

Another might come for the sculpture history.

Another might come because the whole thing is a little weird.

Those are all valid entrances.

What if We Take Air Resistance Seriously?

The Double It and Square It Rule is clean because we ignored air resistance.

But tennis balls are fuzzy, light, and draggy. So a natural next question is:

> How wrong is the rule?

That question is one of the reasons I like this example. A simple classroom demonstration naturally leads to a more advanced mathematical model.

Now we need differential equations.

For the air-resistance model, assume quadratic drag. The drag force points opposite the velocity, so a vertical model is

$$m\frac{dv}{dt} = -mg - \frac{1}{2}\rho C_D A\, v|v|.$$

Dividing by m, we write this as

$$\frac{dv}{dt} = -g - Kv|v|,$$

where

$$K = \frac{\rho C_D A}{2m}.$$

In the model used for the webpage,

$$g = 32, \qquad K \approx 0.00615.$$

Here g is measured in feet per second squared, velocity is measured in feet per second, and height is measured in feet.

Solving the Upward Motion With Drag

On the way up, $v > 0$, so

$$v|v| = v^2.$$

The velocity equation becomes

$$\frac{dv}{dt} = -g - Kv^2.$$

This separates:

$$\frac{dv}{g + Kv^2} = -dt.$$

Integrating gives

$$\arctan\left(v\sqrt{\frac{K}{g}}\right) = q - \sqrt{gK}\,t,$$

where

$$q = \arctan\left(v_0\sqrt{\frac{K}{g}}\right).$$

Let

$$w = \sqrt{gK}.$$

Then the upward velocity can be written as

$$v(t) = \sqrt{\frac{g}{K}}\tan(q - wt).$$

The ball reaches the top when $v(t) = 0$. Since $\tan(0) = 0$, this occurs when

$$q - wt_{\text{top}} = 0.$$

Therefore

$$t_{\text{top}} = \frac{q}{w}.$$

To find the maximum height, integrate velocity:

$$h(t) = \int_0^t v(s)\,ds.$$

Using the formula for $v(t)$, this gives

$$h(t) = \frac{1}{K}\ln\left(\frac{\cos(q - wt)}{\cos q}\right).$$

At the top, $t = q/w$, so $q - wt = 0$. Therefore

$$h_{\max} = \frac{1}{K}\ln\left(\frac{1}{\cos q}\right).$$

Equivalently,

$$h_{\max} = \frac{1}{K} \ln(\sec q).$$

Matching the Same Catch Time

There is one more step.

In the no-drag model, if we know the total catch time T, we immediately get the height.

With drag, the upward and downward parts are not symmetric. The ball loses energy to air resistance, so the downward trip is not just a mirror image of the upward trip.

The time to the top is

$$\frac{q}{w}.$$

For the downward trip from the top back to the starting height, the falling time is

$$\frac{\operatorname{arcosh}(\sec q)}{w}.$$

So the total throw-to-catch time is

$$T_{\text{catch}} = \frac{q + \operatorname{arcosh}(\sec q)}{w}.$$

In practice, if we measure T_{catch}, we solve this equation numerically for q. Then we use

$$h_{\max} = \frac{1}{K} \ln(\sec q)$$

to estimate the maximum height with tennis-ball drag.

This is a nice moment pedagogically because the original problem started in precalculus and calculus, but it naturally leads toward differential equations and numerical methods.

How Good Is the Shortcut?

For the actual George Washington toss, the catch time was about

$$T_{\text{catch}} = 2.83 \text{ seconds}.$$

The no-drag rule gives

$$(2T)^2 \approx 32.04 \text{ feet above my hand.}$$

The tennis-ball drag model gives about

$$32.01 \text{ feet.}$$

That is only about 0.25 inches lower.

So for this toss, air resistance barely changes the estimate.

Here is the comparison table from the webpage model:

Catch time T sec	Double/square ft	Drag model ft	Error ft	Error inches
1.00	4.00	4.00	-0.00	-0.00
2.00	16.00	16.00	-0.00	-0.03
2.83	32.04	32.01	-0.02	-0.25
3.00	36.00	35.97	-0.03	-0.35
4.00	64.00	63.84	-0.16	-1.96
5.00	100.00	99.38	-0.62	-7.38
6.00	144.00	142.21	-1.79	-21.49
7.00	196.00	191.66	-4.34	-52.08

The rule is within about 6 inches until around $T = 4.83$ seconds, which corresponds to a no-drag estimate of about 93 feet.

It is within about 1 foot until around $T = 5.43$ seconds, about 118 feet.

It is within about 2 feet until around $T = 6.12$ seconds, about 150 feet.

Past that, the exponent really wants to be slightly less than 2, but then we lose the beautiful simplicity of Double It and Square It. For sculpture-scale tosses, the simple rule is usually off by inches.

That is good enough for a classroom rule of thumb.

A Practical Ceiling

A tennis racket can hit a tennis ball much harder than I can throw one.

The fastest recorded tennis serves are around 240 ft/sec. If a tennis ball could be hit straight upward at that speed with no air resistance, then the no-drag model would predict a flight time of about

$$T = \frac{2v_0}{g} = \frac{2(240)}{32} = 15 \text{ seconds.}$$

The corresponding no-drag maximum height would be

$$h_{\max} = \frac{v_0^2}{2g} = \frac{240^2}{64} = 900 \text{ feet.}$$

But a tennis ball has significant drag. In the quadratic-drag model, a straight-up hit at that speed stays in the air for only about 7.2 seconds and reaches roughly 200 feet.

That means $T = 7$ seconds is already near a practical ceiling for this page. If a world-class tennis serve aimed straight upward only lasts a little over 7 seconds in the drag model, then my underhand sculpture tosses are not going to produce anything much longer than that.

Why This Changed My Teaching

The most important thing about this example is not the rule.

The most important thing is what happened to me while making it.

I started with something I had done many times before: throw a tennis ball and talk about vertical motion. But then I made one small decision. I decided to take the tennis ball outside and connect it to something real.

That decision changed the problem.

Suddenly I was not only teaching a formula. I was measuring a campus sculpture. I was reading about public art. I was thinking about George Washington's head, which is not a sentence I expected to write in a calculus project. I was connecting height, velocity, history, video, graphing, and a rule of thumb that was hiding inside a standard projectile-motion equation.

That was the real lesson for me.

There was something new to learn in a topic I had taught dozens of times.

I just had to be curious again.

After that, I started looking differently at everything I taught. Every topic had a hidden story, a strange application, an overlooked visual, a historical connection, or a better way in. Some topics only needed a question. Some needed a bad joke. Some needed a Desmos animation. Some needed me to embarrass myself in public with a tennis ball.

That is not a bad trade.

What I Hope Students Carry

I hope students remember the tennis ball.

I hope they remember that velocity is the slope of height.

I hope they remember that acceleration can be negative even while the ball is moving upward.

I hope they remember that derivatives help us find the bumps.

I hope they remember that a simple model can be useful even when it is not perfect.

I hope they remember that assumptions matter.

I hope they remember that the campus itself can be part of the classroom.

And maybe, when they walk past a sculpture, or throw a ball, or watch something move, they might think:

> There is a graph here. There is a story here. I can understand this.

That is the durable anchor.