How Geometry Shapes the Feeling of a Roller Coaster

Hannah · Spring 2026

Idea Taxonomy

The project is organized around a chain of ideas connecting track geometry to what riders feel:

curve	-> k & t ->	acceleration	->	jerk	->	ride comfort

Level 1 — Basic Shape

A parametric space curve **r**(t) = (X(t), Y(t), Z(t)) describes the full 3D path of the track. Everything — forces, speed, comfort — flows from the geometry of this curve.

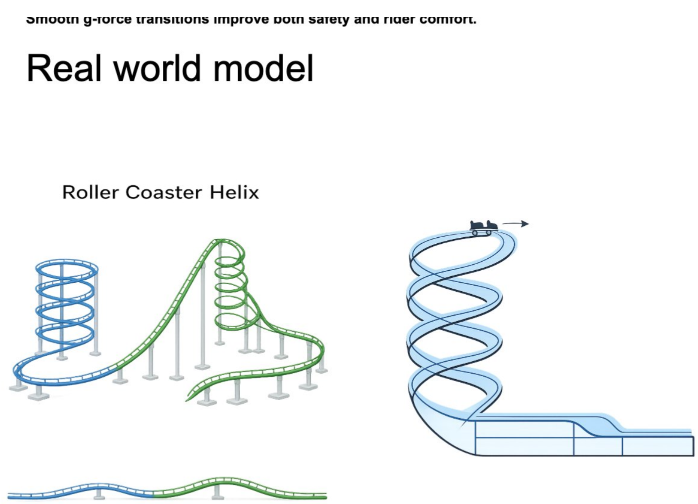


Fig. 1 — Real-world roller coaster helix examples. The blue cylindrical helix and the green coaster with lift hill both use helical segments where torsion tau != 0, producing the twisting 3D path visible above.

Level 2 — Curvature k

Curvature k = 1/R measures how sharply the track bends. It connects directly to the force riders feel:

```
a_N = v^2 / R = v^2 * k
```

- Constant k -> circle -> steady centripetal force
- k = 0 -> straight line -> no lateral force
- Abrupt jump in k -> sudden spike in a_N -> jolt
- Gradually changing k (clothoid) -> smooth acceleration -> comfortable ride

Full curvature formula:

```
k = ||r'(t) x r''(t)|| / ||r'(t)||^3
```

Circular Loop vs. Clothoid Loop (Euler Spiral)

This is the central comparison of the project:

Circular Loop

- Curvature jumps suddenly from 0 (straight) to 1/R (circle)
- a_N = k*v^2 spikes instantly -> abrupt jerk -> uncomfortable
- G-force graph shows a sharp discontinuous jump at loop entry

Clothoid Loop (Euler Spiral)

- Curvature increases gradually with arc length: k(s) = s / A^2
- Smooth increase in a_N -> low jerk -> comfortable transition
- G-force graph shows a gradual ramp instead of a sudden spike

Key Insight: Smooth curvature transitions produce smoother acceleration and more comfortable roller coaster motion.

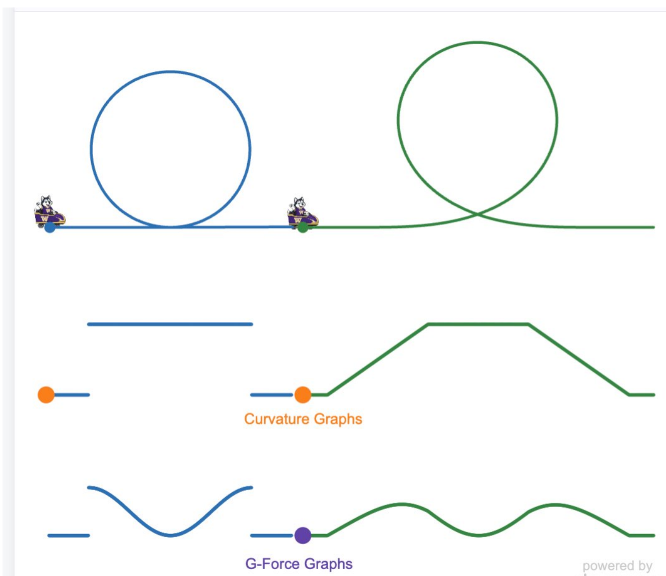


Fig. 2 — Curvature graphs (middle row) and G-force graphs (bottom row) comparing the circular loop (blue, left) and the clothoid loop (green, right). The blue curvature graph shows an abrupt flat line; the green shows a smooth ramp.

G-Force and Transition Length

Felt g-force is based on apparent acceleration:

$$G = ||a - g|| / g$$

For a vertical circular loop:

$$G_{bottom} = 1 + v^2/(R*g) \;|\; G_{top} = v^2/(R*g) - 1$$

Since $k = 1/R$, g-force is proportional to curvature:

$$G = (v^2 * k) / g \rightarrow k \text{ up means } G \text{ up}$$

Effect of Transition Length L

Comparing designs with different clothoid transition lengths reveals the trade-off between smoothness and physical space:

Design	L	Smoothness	Max G-force	Space Needed
Circular	0	Worst	High spike	Smallest
Short	0.5*pi*R	Better	Less spike	Medium
Medium	pi*R	Smooth	Controlled	Balanced
Long	1.5*pi*R	Smoothest	Controlled	Largest

A medium transition length gives the best balance between comfort, thrill, and compactness.

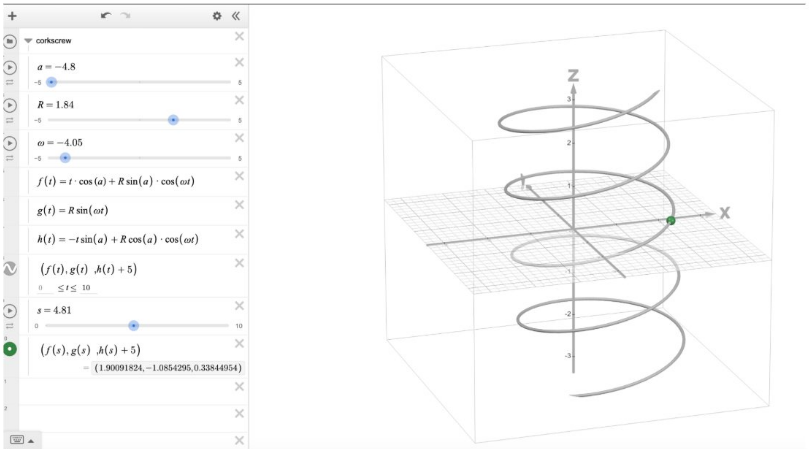
Level 3 — Torsion tau

Torsion tau measures how the curve twists out of its osculating plane. While curvature keeps a curve within a plane, torsion lifts it into 3D — essential for helices, corkscrews, and banked turns:

$$dB/ds = -\tau * N \rightarrow \tau = (r' \times r'') \cdot r''' / ||r' \times r''||^2$$

- tau = 0 -> planar curve (circle, Euler spiral)
- tau != 0 -> curve twists through space (helix, corkscrew, banked return)
- tau is the magnitude of the change in the binormal vector B

Torsion is what makes a roller coaster truly three-dimensional.



https://www.desmos.com/3d/zfudteijee?lang=zh-CN

*Fig. 3 — Desmos 3D corkscrew visualization. The parametric equations f(t) = t*cos(a) + R*sin(a)*cos(wt), g(t) = R*sin(wt), h(t) = -t*sin(a) + R*cos(a)*cos(wt) produce a helical corkscrew where both k != 0 and tau != 0.*

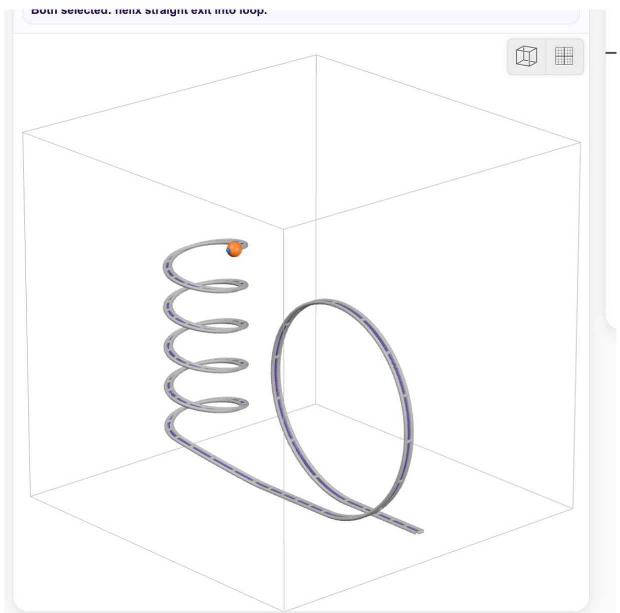


Fig. 4 — Full coaster: descending helix (tau != 0) connecting to a vertical loop (high k). The helix demonstrates sustained torsion; the loop shows the curvature spike.

The Frenet Frame (T, N, B)

The Frenet-Serret frame is a moving coordinate system that travels with the curve. Together k and tau describe exactly how this frame rotates:

- **T** (Tangent) — points in the direction of motion
- **N** (Normal) — points toward center of curvature; riders feel force in this direction
- **B** (Binormal) = T x N — points sideways, out of the osculating plane

Curve Type	k	tau	What You Feel
Straight line	0	0	No lateral force
Circle	!= 0	0	Steady sideways push
Euler spiral	increases	0	Gradually increasing force
Helix	!= 0	!= 0	Push + twisting sensation
Wavy helix	changes	changes	Complex 3D forces
Banking section	!= 0	!= 0	Lateral force into seat

Track Segments via the Frenet Frame

Track Segment	k / tau Behavior	Rider Experience
Straight	k = 0, tau = 0	No lateral forces
Clothoid entry	k increases, tau = 0	Forces build smoothly (low jerk)
Circular arc	k = constant, tau = 0	Stable centripetal force
Banking section	tau != 0	Lateral force redirected into seat
Clothoid exit	k decreases to 0	Smooth transition out

Track construction: straight -> clothoid (entry) -> curve (circle / helix) -> banking (torsion) -> clothoid (exit)

Key Takeaways

Curvature controls the jolt. Any abrupt jump in k becomes a spike in a_N = k*v^2. Smooth k(s) means smooth forces.

Torsion adds the third dimension. Without tau, every track section stays in a plane. Nonzero tau produces helices, corkscrews, and banks.

Speed amplifies everything. Because a_N = k*v^2, doubling speed quadruples the centripetal force. Height is the speed multiplier.

The clothoid is the optimal transition. The Euler spiral is the unique curve where k grows linearly with arc length, producing constant jerk.

Calculus is the design language. The TNB frame, k, tau, and energy conservation are the tools real engineers use to build safe rides.

Reference: Pombo & Ambrósio — Modelling Tracks for Roller Coaster Dynamics, 2014

Interactive visualizations: Desmos 3D · Math 126 — Spring 2026