How Shape Becomes Sound: Investigation on Guitar Acoustics

Tessa Xu

Introduction

Most people think that a guitar's sound comes from its strings; however, the strings alone produce very little sound. The rich and recognizable tone of a guitar comes from a sequence of physical processes that transform a simple vibration into a complex acoustic signal. This observation motivates the central question of this project:

> How does the shape of a guitar influence the sound it produces?

To answer this question, we follow the journey of sound from a plucked string, through a resonant cavity, and finally into the surrounding air.

Act I: A String is Plucked

When a guitarist plucks a string, elastic energy stored in the stretched string is released and converted into vibration. Rather than oscillating at a single frequency, the string vibrates as a superposition of many standing-wave modes. These vibrations can be represented using a Fourier series:

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi ct}{L}\right),$$

where L is the string length, c is the wave speed, and A_n determines the strength of the n-th harmonic.

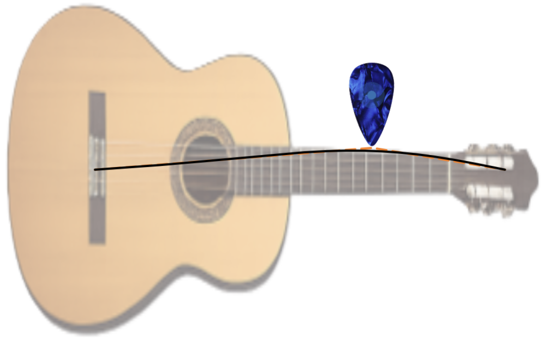

Figure 1: A plucked guitar string.

The position at which the string is plucked affects the coefficients A_n. Plucking near the center emphasizes lower harmonics, while plucking closer to the bridge excites stronger high-frequency components.

Act II: The Guitar Body as an Amplifier

Through the bridge, the string excites the wooden soundboard and the enclosed air cavity inside the instrument. Together, these components act as a resonant system, which does not amplify all frequencies equally.One way to model this behavior is with a resonance filter:

$$H(f) = \exp\left(-\left(\frac{f - f_0}{w}\right)^2\right),$$

where f_0 represents a preferred resonance frequency and w controls the resonance bandwidth.

Act III: Waves Inside a Resonant Cavity

To understand why different guitar bodies sound different, we look inside the resonant cavity itself. Sound waves generated by the vibrating soundboard reflect repeatedly from the cavity walls.To study this phenomenon, we first consider a simplified rectangular cavity model.

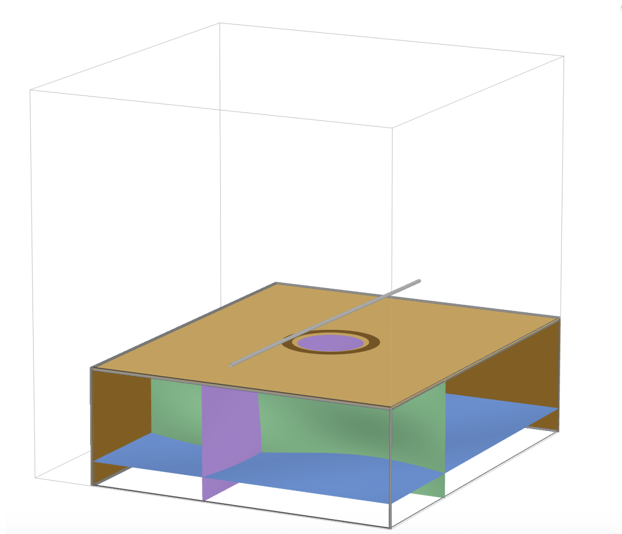

Figure 2: A simplified cavity model showing a standing-wave pattern inside an enclosed volume.

Even in this simple geometry, wave motion is far from random. Certain vibration patterns appear naturally, while others cannot exist. These preferred vibration patterns are called resonance modes. The existence of resonance modes suggests a deeper question:

> How does geometry determine which wave patterns are possible?

Act IV: Eigenmodes and Resonance

The answer is provided by the Helmholtz equation,

$$\nabla^2 u + \lambda u = 0,$$

Solutions to this equation are known as eigenmodes. Each eigenmode represents a natural standing-wave pattern of the cavity, while each eigenvalue corresponds to a resonance frequency. The shape of the cavity determines both the eigenmodes and the eigenvalues.

Act V: The Shape of a Guitar

Real guitars possess far more complicated geometries than simple rectangular boxes. As a result, their resonance patterns become significantly more complex.

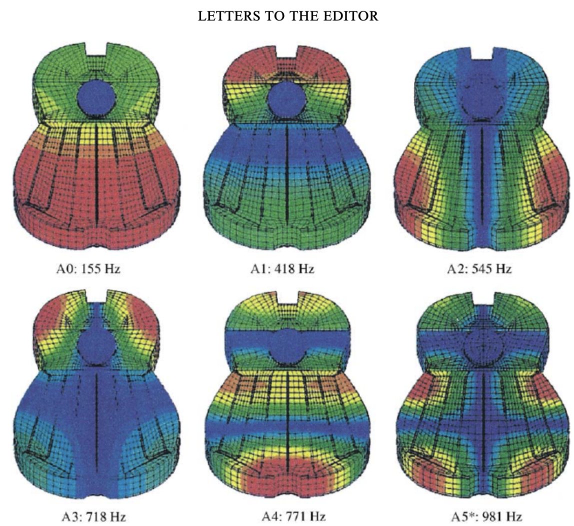

Figure 3: Example modal shapes of a guitar body at several resonance frequencies.

The modal shapes shown above illustrate how different regions of the instrument move at different resonance frequencies. At low frequencies, large portions of the guitar move together. At higher frequencies, the vibration patterns become increasingly intricate.

These resonance patterns influence which frequencies are amplified most strongly and therefore contribute directly to the instrument's timbre. The sound of a guitar is therefore not determined solely by its strings. It is also determined by the geometry of the structure supporting those strings.

Act VI: Helmholtz Resonance

Not all resonances arise from standing-wave patterns distributed throughout the cavity. Another important phenomenon is Helmholtz resonance, which is similar to the sound produced when blowing across the top of a bottle. The corresponding resonance frequency is approximately

$$f_H = \frac{c}{2\pi}\sqrt{\frac{A}{VL}},$$

where

- A is the sound-hole area,
- V is the cavity volume,
- L is an effective neck length.

This resonance primarily influences low-frequency response and contributes to the characteristic warmth and fullness of a guitar's sound. Together, Helmholtz resonance and cavity eigenmodes create the acoustic fingerprint of the instrument.

Conclusion

A vibrating string generates harmonics. These harmonics excite the guitar body and enclosed air cavity. The cavity supports resonant modes determined by its geometry, selectively amplifying some frequencies while suppressing others. The resulting sound is therefore shaped not only by the string itself but also by the mathematical structure of the space surrounding it.

By studying standing waves, resonance filters, Helmholtz resonance, and cavity eigenmodes, we gain insight into how physical shape becomes musical sound.

More broadly, this project connects to one of the most famous questions in mathematical physics:

> Can one hear the shape of a drum?

$$\text{Shape} \to \text{Eigenmodes} \to \text{Resonance} \to \text{Sound.}$$

The geometry of an instrument therefore leaves a measurable signature on the waves it supports. In a guitar, we hear this principle every time a note is played. Through this project, we see how mathematics provides a bridge between physical shape, wave behavior, and musical acoustics.