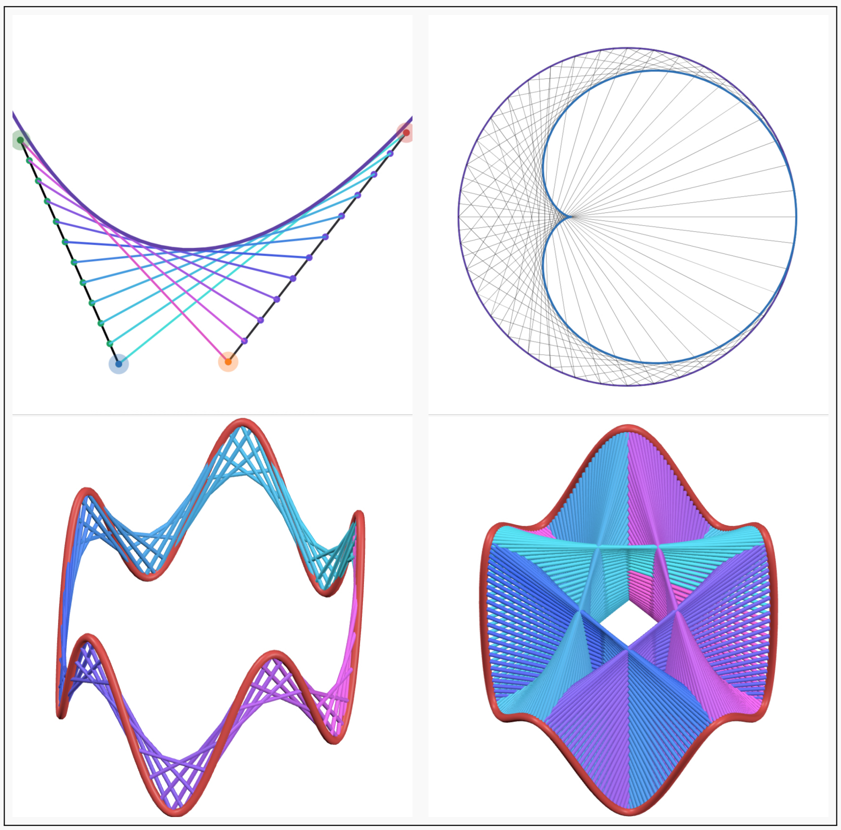
0.1 Marcus 2 - Weaving Curves

The Idea. What shapes, curves, and envelopes arise when you connect two curves to each other with equally spaced lines?

The Story / Why we care. This project began by connecting two straight lines on equally spaced intervals and noting the shape (envelope) formed by the connecting lines. Formulas have been derived for the 2D envelopes traced by lines between two curves. We moved on to connecting parameterized 3D closed loops with themselves and noting the surfaces that formed. We tried applying the 2D formulas to this situation, but were unable to get any useful information out of them. Using Mathematica, we generated STLs of curves and their connecting lines

Visual.



Try this.

- How does changing the time offset of a curve affect the connections?
- Do you think the 3D surfaces can be represented algebraically?

Interactive.

- Desmos links:
  - 2D Curve, Cartoid, 3D Curve

The Mathematics.

A curve in $\mathbb{R}^3$ can be represented parametrically by $A(t) = (f(t), g(t), h(t))$ where (x, y, z) changes with respect to t. By setting each of these functions to be composed of variations on $\sin(2\pi t)$ and $\cos(2\pi t)$ and having $0 \leq t \leq 1$, it ensures that the curve is a closed loop.

Let M be a list of N_t elements. $\frac{M}{N_t}$ represents N_t equally spaced numbers from $0 \to 1$.

By plugging $t = \frac{M}{N_t}$ into our function $A(t)$, we get N_t equally time-seperated points in $\mathbb{R}^3$ along the curve $A(t)$.

Let s be the "time offset." That is, how much "further along" one point is relative to the others. $A\left(\frac{M}{N_t} + s\right)$ is s ahead of $A\left(\frac{M}{N_t}\right)$.

To make a parameterized line from one point to another in $\mathbb{R}^3$, use the formula $(x_1, y_1, z_1) + t(x_2 - x_1, y_2 - y_1, z_2 - z_1)$ where $0 \leq t \leq 1$. Plugging in our points from $A(t)$ and $A(t+s)$, we get:

$$\boxed{A\left(\frac{M}{N_t}\right) + t\left(A\left(\frac{M}{N_t} + s\right) - A\left(\frac{M}{N_t}\right)\right)}$$

Since this equation contains the list M, it plots N_t lines. As $N_t \to \infty$, the surface becomes apparent.
Changing the value of s between $0 < s \leq \frac{1}{2}$ yields different surfaces.

How to use this.

- Parameterization is used here to represent the curve in $\mathbb{R}^3$ and the lines from $A\left(\frac{M}{N_t}\right)$ to $A\left(\frac{M}{N_t} + s\right)$
- Parameterization is a useful tool to break complex functions into easier-to-work-with variables
- Parameterization is introduced in Calculus III and is heavily expanded on in Calculus IV

Questions to explore.

- What happens if $A(t)$ is not a closed curve?
- Is there a way to algebraically represent the surface formed as $N_t \to \infty$?

Connections.

- In $\mathbb{R}^2$, the determinant (det) is used to know when two vectors are orthogonal to each other ($\perp$)
- The determinant (det) is introduced in Linear Algebra

Notes / Further reading.

- For more details, see the supplementary document:
  - On a Tangent
- Interactive Webpage